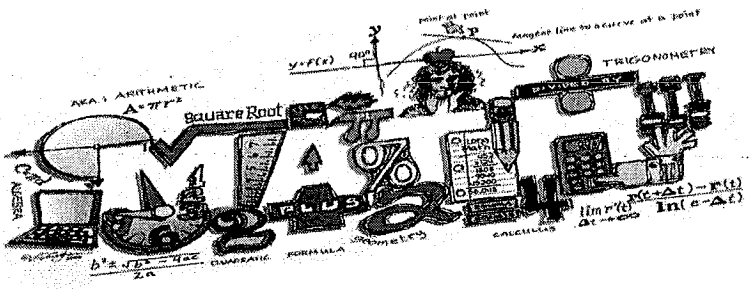
If AVAILABLE: Students should complete 30 minutes of ALEKS daily too!

Email your teacher & Mr. Sayers for log-in info from 9:30 – 11am or 1pm-2:30 pm
scott.sayers@richlandone.org, naytoniyan.green@richlandone.org
Kevin.fletcher@richlandone.org, angela.williams@richlandone.org

All assignments will be GRADED! Please be sure to submit your completed packet NO LATER THAN 5 days upon returning to school.



7th Grade Suggested Timeline

Technology-Free Alternative

Days	Activity Math Essentials Workbook Read all the items in the margins and complete the activities as directed.
1	Pages 153-157
2	Pages 159-162
3	Pages 170-175
4	Pages 177-181
5	Pages 183-186
6	Pages 188-190
7	Pages 192-193
8	Pages 192-193
9	Pages 199-203
10	Pages 205-209
11	Pages 211-216
12	Pages 218-224
13	Pages 226-232
14	Pages 234-239
15	Pages 241-244
16	Pages 246-252
17-18	Pages 254-255
19-20	Pages 264-265

Never Stop Learning

Name _____

Construct Geometric Shapes

Common Core State Standard: 7.G.2

Getting the Idea

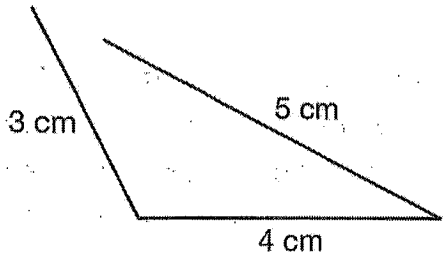
Triangles can be constructed using simple tools such as a ruler and protractor, or using more complex tools such as computer drawing technology. When drawing triangles with a ruler and/or a protractor by hand, you may need to erase your work and start over again sometimes. It can involve some trial and error.

Example 1

Using a ruler, construct a triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters. What kind of triangle is it? Is it possible to draw another kind of triangle?

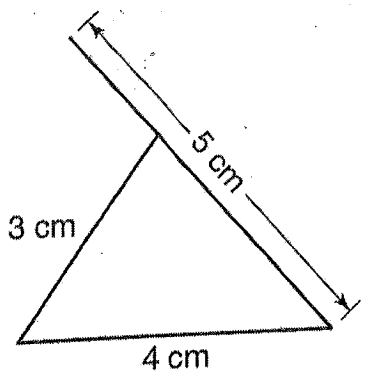
Strategy **Use a ruler.**

Step 1 Try drawing a triangle with one obtuse angle—an obtuse triangle.



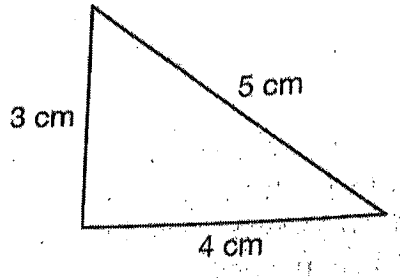
The figure is not closed. An obtuse triangle is not possible.

Step 2 Try drawing a triangle with only acute angles—an acute triangle.



The endpoints do not meet. An acute triangle is not possible.

Step 3 Try drawing a triangle with one right angle—a right triangle.



You can draw one unique triangle with those side lengths, and it is a right triangle.

Solution **It is only possible to draw a right triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters.**

When trying to construct a triangle with given side lengths or angle measures, there are several possibilities:

- The triangle may be uniquely defined. In other words, you may only be able to draw one triangle.
- The triangle may be ambiguously defined. That just means you may be able to draw more than one triangle.
- The triangle may be nonexistent. It may not be possible to draw a triangle with those measures.

The triangle in Example 1 is uniquely defined.

Below is a theorem that can help you construct triangles with given angle measures.

Triangle Angle Sum Theorem

This theorem states that the sum of the angles in any triangle is 180°.

Example 2

Is it possible to construct a triangle with angles measuring 61°, 33°, and 86°? If so, can you only draw one unique triangle or can you draw many different triangles?

Strategy **Use the triangle angle sum theorem to test the angle measures. Then decide if it is possible to draw one unique triangle or many different triangles.**

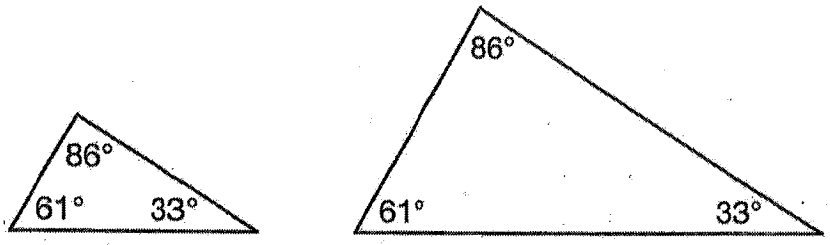
Step 1 Determine if the angle measures add to 180°.

$$61° + 33° + 86° \stackrel{?}{=} 180°$$

$$180° = 180° \quad ✓$$

So, a triangle with these angle measures is possible.

Step 2 Use a protractor to draw one or more triangles with those angle measures.



The two triangles above have the correct angle measures, but they have different lengths.

That is because triangles with the same angle measures are similar to one another.

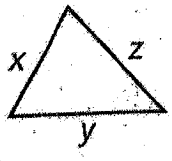
So, it is possible to draw many different triangles with those angle measures.

Solution **A triangle with angles measuring 61°, 33°, and 86° is ambiguously defined because no side lengths are mentioned. It is possible to draw many different similar triangles with those angle measures.**

Another theorem that may help you when constructing triangles is given below.

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$x + y > z$ $x + z > y$ $y + z > x$

Example 3

Can you construct a triangle with sides measuring 5 inches, 8 inches, and 15 inches?

Strategy **Use the triangle inequality theorem.**

Step 1 Use the triangle inequality theorem.

If this triangle is possible, the sum of any two side lengths should not be greater than the third side length.

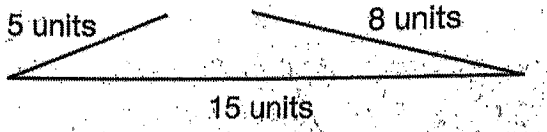
$5 + 8 \stackrel{?}{>} 15$?

$13 < 15$, so the inequality $5 + 8 > 15$ is not true.

It is impossible to draw a triangle with those side lengths.

Step 2 Try to sketch a triangle with those dimensions so you can see why it is not possible.

Use units other than inches to keep your drawing a manageable size.



There is no way to connect all three sides. It is impossible to draw a triangle with those side lengths.

Solution **A triangle with sides measuring 5 inches, 8 inches, and 15 inches does not exist.**

Coached Example

Is it possible to construct a triangle with sides measuring 8 feet, 9 feet, and 12 feet?

The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle must be _____ than the length of the third side.

Use the theorem to determine if this triangle is possible or not.

Is $8 + 9 > 12$? _____

Is $8 + 12 > 9$? _____

Is $9 + 12 > 8$? _____

The inequalities above are all true, so it _____ possible to draw a triangle with side lengths of 8 feet, 9 feet, and 12 feet.

Lesson Practice

Choose the correct answer.

1. Which best describes a triangle with side lengths of 6 inches, 8 inches, and 9 inches?

   A. ambiguously defined
   B. nonexistent
   C. a unique, acute triangle
   D. a unique, right triangle

2. Which best describes a triangle with angles measuring 60°, 40°, and 100°?

   A. ambiguously defined
   B. nonexistent
   C. a unique, acute triangle
   D. a unique, right triangle

3. Which best describes a triangle with side lengths of 3 inches, 4 inches, and 8 inches?

   A. ambiguously defined
   B. nonexistent
   C. a unique, obtuse triangle
   D. a unique, right triangle

4. Which best describes a triangle with side lengths of 5 centimeters, 12 centimeters, and 13 centimeters?

   A. ambiguously defined
   B. nonexistent
   C. a unique, acute triangle
   D. a unique, right triangle

5. Lincoln constructs a triangle with one side 5 inches long and another side 7 inches long. Which is **not** a possible length for the third side?

   A. 3 inches
   B. 6 inches
   C. 11 inches
   D. 12 inches

6. Maggie constructs a triangle with one side 7 centimeters long and another side 10 centimeters long. Which is **not** a possible length for the third side?

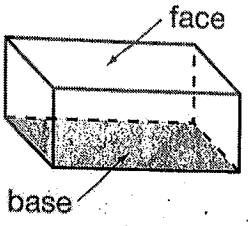
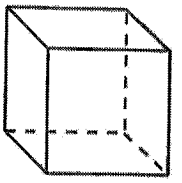
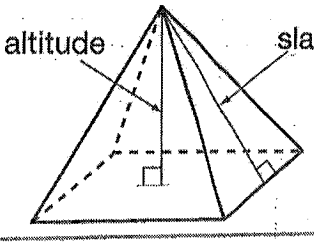
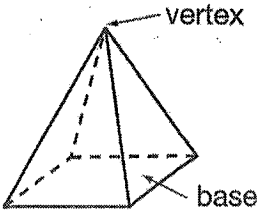
   A. 2 centimeters
   B. 4 centimeters
   C. 7 centimeters
   D. 16 centimeters

Cross Sections of Three-Dimensional Figures

Common Core State Standard: 7.G.3

Getting the Idea

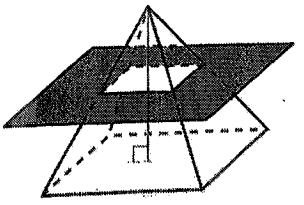
A **three-dimensional figure** (also called a **solid figure**) has length, width, and height. It is not flat. Some examples of three-dimensional figures are below.

A **prism** has a pair of bases that are parallel, congruent polygons. Its other faces are rectangles.	A **rectangular prism** has 6 faces that are rectangles. 	A **cube** is a rectangular prism with 6 square faces. 
A **pyramid** has one base that is a polygon. Its other faces are triangles. The height of a pyramid is called its **altitude**, and the height of its lateral face is called its **slant height**.	A **rectangular pyramid** has a base that is a rectangle. 	A **square pyramid** has a base that is a square. 

A three-dimensional figure can be sliced by a plane to show a two dimensional view. This view is called a **cross section**.

Example 1

A square pyramid is sliced by a plane that is parallel to its base, as shown.



What is the shape of the cross section?

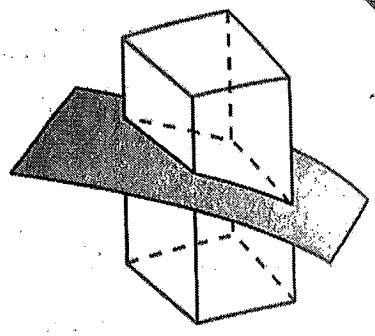
Strategy **Visualize a plane, parallel to the base, slicing through the pyramid.**

The cross section will have the same shape as the base.

It will be a square.

Solution **The shape of the cross section is a square.**

Example 2



A rectangular prism is cut by the slanted plane shown.

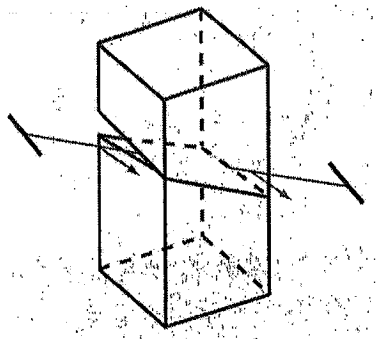
What is the shape of the cross section?

Strategy **Visualize the prism being sliced by a thin piece of wire.**

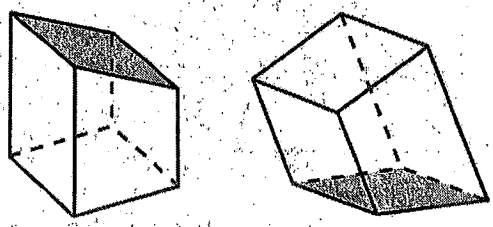
Step 1 Determine the angle at which the plane intersects the prism.

The plane is neither horizontal nor vertical to the faces of the prism.

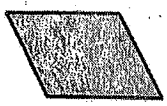
Step 2 Imagine slicing the rectangular prism with a piece of wire.



Step 3 The prism is now in two parts.



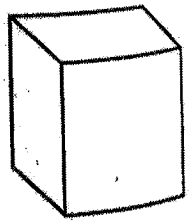
Step 4 Imagine viewing the cut prism from above and from the side.



Solution **The shape of the cross section is a parallelogram.**

Example 3

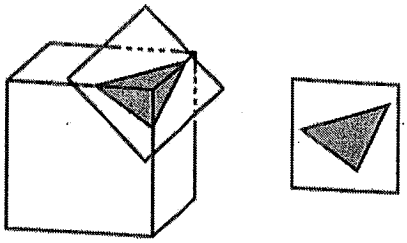
Look at this cube.



How can a plane slice the cube so that the cross section is a triangle?

Strategy **Visualize using a plane to slice the cube to get a triangular cross section.**

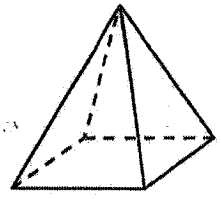
Slice through a corner of the cube with a plane.



Solution **The cross section of the cube is shown above.**

Coached Example

Nari will slice this pyramid with a plane that is perpendicular to the base and passes through the top vertex.



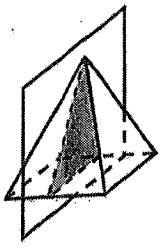
What is the shape of the cross section?

What does "perpendicular" mean?

_____

_____

Visualize slicing the prism with a plane that is perpendicular to the base and passes through the top vertex.



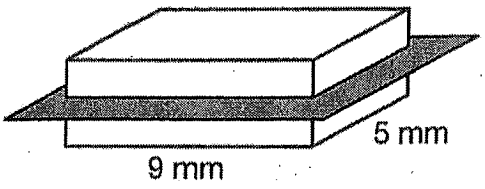
Make a sketch of the cross section in the space below.

The shape of the cross section is _____.

Lesson Practice

Choose the correct answer.

1. The rectangular prism is being sliced by a plane parallel to its base.



What will be the shape of the cross section formed?

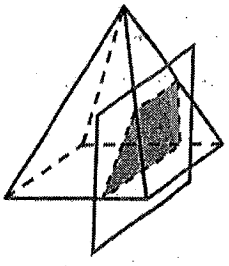
A. parallelogram that is not a rectangle

B. rectangle that is not a square

C. square

D. triangle

2. What is the shape of the cross section formed when the square pyramid is sliced by a plane perpendicular to its base that does **not** pass through its top vertex?



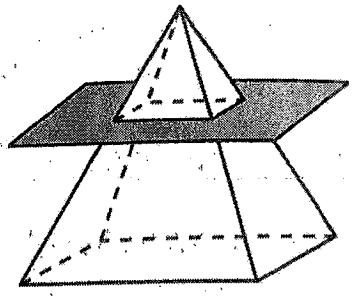
A. parallelogram (not a square)

B. square

C. trapezoid

D. triangle

3. What is the shape of the cross section formed when the rectangular pyramid is cut by the plane parallel to its base?



A. 

B. 

C. 

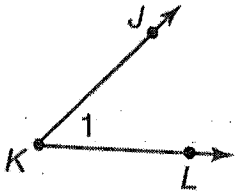
D. 

Angles

Common Core State Standard: 7.G.5

Getting the Idea

An **angle** is a geometric figure formed by two rays that have a common endpoint called the **vertex**. The angle below can be named ∠1, ∠*JKL*, ∠*LKJ*, or ∠*K*.



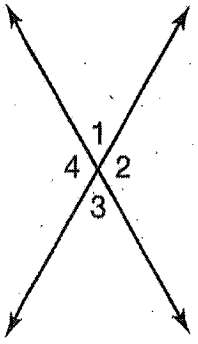
Angles are measured in degrees (°) and can be classified by their angle measures.

An **acute angle** is an angle that measures less than 90°.

A **right angle** is an angle that measures 90°.

An **obtuse angle** is an angle that measures greater than 90° and less than 180°.

Intersecting lines can form some special angle pairs.



Adjacent angles are two angles with a side in common.
∠1 and ∠2 are one set of adjacent angles.

Vertical angles are two non-adjacent angles, formed by intersecting lines, and are congruent.
∠1 and ∠3 are one set of vertical angles.

Supplementary angles are two angles whose measures have a sum of 180°.
Supplementary angles that are adjacent angles form a **straight angle.**
∠4 and ∠3 are one set of supplementary angles.

Complementary angles are two angles whose measures have a sum of 90°.

To write the measure of an angle, you can use the abbreviation, m.

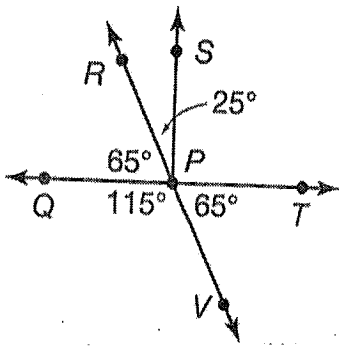
For example, "the measure of angle *x*" can be written as "m∠*x*."

Example 1

Look at the diagram on the right.

Find the following:

- a pair of complementary angles
- a pair of supplementary angles
- a pair of adjacent angles
- a pair of vertical angles



Strategy **Use the definitions to identify the angle pairs.**

Step 1 Find a pair of complementary angles.

Find two angles that have a sum of 90°.

$m\angle QPR = 65°$ and $m\angle RPS = 25°$.

$65° + 25° = 90°$

Step 2 Find a pair of supplementary angles.

Find two angles that have a sum of 180°.

$m\angle QPV = 115°$ and $m\angle TPV = 65°$.

$115° + 65° = 180°$

Note: Another pair of supplementary angles is $\angle QPR$ and $\angle QPV$.

Step 3 Find a pair of adjacent angles.

Find two angles with a side in common.

$\angle QPR$ and $\angle RPS$ have $\overrightarrow{PR}$ in common.

Note: Other pairs of adjacent angles are $\angle RPS$ and $\angle SPT$, $\angle SPT$ and $\angle TPV$, $\angle TPV$ and $\angle QPV$, and $\angle QPV$ and $\angle QPR$.

Step 4 Find a pair of vertical angles.

Find two non-adjacent angles formed by intersecting lines.

$\overleftrightarrow{RV}$ and $\overleftrightarrow{QT}$ intersect at point P to form $\angle QPR$ and $\angle TPV$.

These angles are vertical angles, and they are congruent.

Solution **$\angle QPR$ and $\angle RPS$ are a pair of complementary angles.**

$\angle QPV$ and $\angle TPV$ are a pair of supplementary angles.

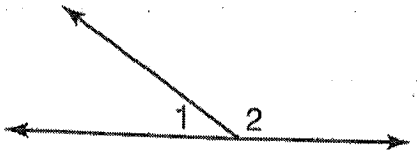
$\angle QPR$ and $\angle RPS$ are a pair of adjacent angles.

$\angle QPR$ and $\angle TPV$ are a pair of vertical angles.

Example 2

In the figure, the measure of $\angle 1$ is 35°.

What is the measure of $\angle 2$?



Strategy **Look for a special angle pair.**

Step 1 Decide what type of angles are $\angle 1$ and $\angle 2$.

The angles are adjacent angles that form a straight line.

So, $\angle 1$ and $\angle 2$ are supplementary angles.

Step 2 What angle measures do you know?

The measure of $\angle 1$ is 35°.

Supplementary angles have a sum of 180°.

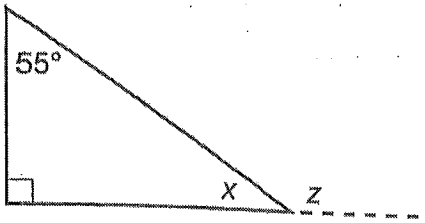
Step 3 Subtract 35° from 180° to find the measure of $\angle 2$.

$180° - 35° = 145°$

Solution **The measure of $\angle 2$ is 145°.**

Example 3

What are the measures of $\angle x$ and $\angle z$?



Strategy **Look for special relationships between angles.**

Step 1 Find the measure of $\angle x$.

The measures of the angles in a triangle have a sum of 180°.

The measures given are 55° and a right angle, which measures 90°.

Write an equation and solve for the measure of $\angle x$.

$$55° + 90° + m\angle x = 180°$$
$$145° + m\angle x = 180°$$
$$145° - 145° + m\angle x = 180° - 145°$$
$$m\angle x = 35°$$

Step 2 Identify the angle relationship of $\angle x$ and $\angle z$.

$\angle x$ is adjacent to $\angle z$. The two angles form a straight line.

Supplementary angles are adjacent angles that form a straight line.

So, $\angle x$ and $\angle z$ are supplementary angles.

Step 3 Recall the definition of supplementary angles.

Supplementary angles have a sum of 180°.

Step 4 Write an equation to find the measure of $\angle z$.

$$m\angle x + m\angle z = 180°$$
$$35° + m\angle z = 180°$$
$$35° - 35° + m\angle z = 180° - 35°$$
$$m\angle z = 145°$$

Solution **The measure of $\angle x$ is 35°. The measure of $\angle z$ is 145°.**

Coached Example

Two angles are complementary angles. One angle measures 59°. The other angle measures $(4n - 1)°$. What is the value of n?

Complementary angles have a measure of _____.

The measure of one angle is _____°, and the measure of the other angle is _____°.

Write an equation for the total of the two angles.

_____ + _____ = 90

Solve the equation for n.

To find the measure of the unknown angle, substitute the value of n into $4n - 1$ and evaluate. _____

Check that the sum of the angle measures is 90°.

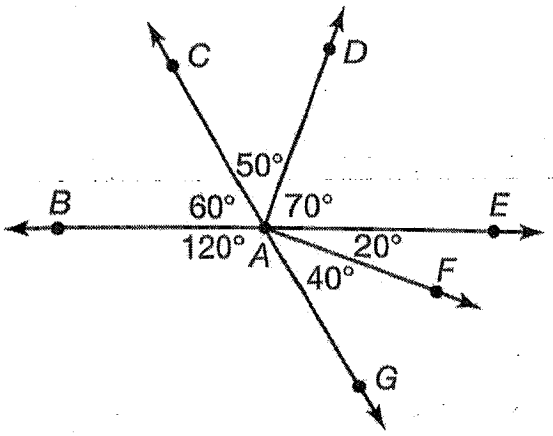
_____

The value of n is _____.

Lesson Practice

Choose the correct answer.

Use the diagram for questions 1–4.



1. Which angle is complementary to $\angle FAE$?

   A. $\angle GAF$
   B. $\angle BAC$
   C. $\angle DAE$
   D. $\angle DAC$

2. Which angle is supplementary to $\angle GAB$?

   A. $\angle BAC$
   B. $\angle DAC$
   C. $\angle DAE$
   D. $\angle EAF$

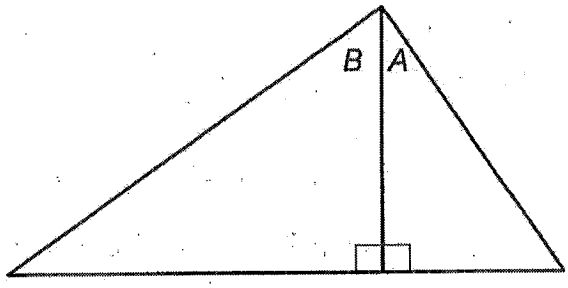
3. Which pair of angles are adjacent angles?

   A. $\angle BAC$ and $\angle GAF$
   B. $\angle DAC$ and $\angle GAF$
   C. $\angle DAE$ and $\angle BAC$
   D. $\angle CAD$ and $\angle DAE$

4. Which pair of angles are vertical angles?

   A. $\angle EAG$ and $\angle BAC$
   B. $\angle GAB$ and $\angle DAF$
   C. $\angle DAC$ and $\angle DAF$
   D. $\angle BAC$ and $\angle CAE$

5. $\angle A$ and $\angle B$ are complementary angles. If $m\angle A$ is 35°, what is the measure of $\angle B$?



   A. 55°
   B. 65°
   C. 90°
   D. 145°

6. $\angle J$ and $\angle K$ are supplementary angles. If $m\angle K$ is 84°, what is the measure of $\angle J$?

   A. 16°
   B. 86°
   C. 96°
   D. 106°

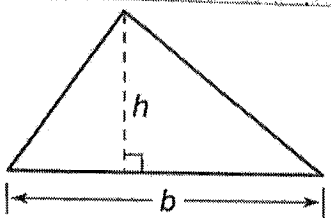
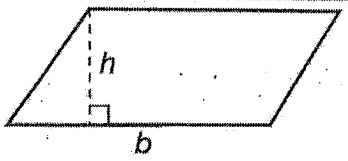
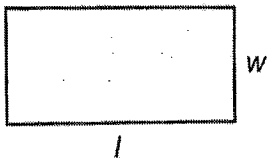
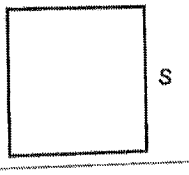
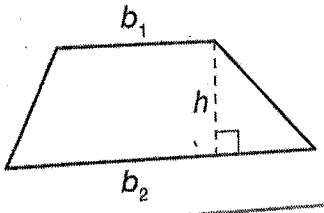
Area

Common Core State Standard: 7.G.6

Getting the Idea

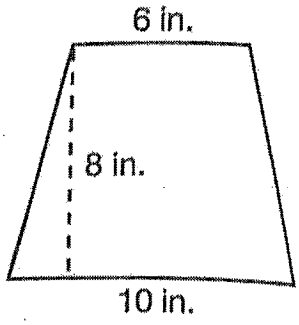
The **area** of a figure is the number of square units inside the figure.

Below are some formulas that can be used to find the areas of common polygons.

Formula	Diagram
Triangle $A = \frac{1}{2}bh$, where b represents the base length and h represents the height.	
Parallelogram $A = bh$, where b represents the base length and h represents the height.	
Rectangle $A = lw$, where l represents the length and w represents the width.	
Square $A = s^2$, where s represents the length of a side.	
Trapezoid $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 represent the base lengths and h represents the height.	

Example 1

What is the area of this trapezoid?



Strategy **Use the formula for the area of a trapezoid.**

Step 1 Write the formula for the area of a trapezoid.

$$A = \frac{1}{2}(b_1 + b_2)h$$

Step 2 Substitute the known values in the formula and simplify.

Let $b_1 = 10$ in. and let $b_2 = 6$ in.

$$A = \frac{1}{2}(10 \text{ in.} + 6 \text{ in.}) \times 8 \text{ in.}$$

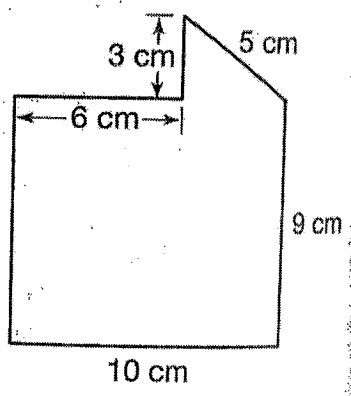
$$= \frac{1}{2}(16 \text{ in.}) \times 8 \text{ in.}$$

$$= 8 \text{ in.} \times 8 \text{ in.} = 64 \text{ in.}^2$$

Solution **The area of the trapezoid is 64 in.^{2}**

Example 2

Phillip drew the figure on the right to represent the design of his new garage. What is the area of the figure?

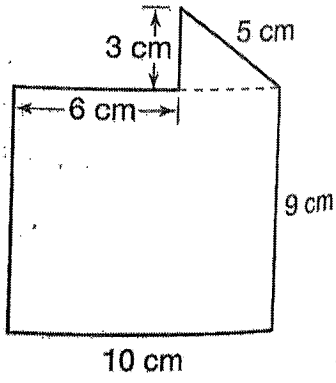


Strategy **Divide the figure into smaller, familiar figures. Find the area of each figure. Then add to find the total area.**

Step 1 Divide the figure into familiar figures.

The figure is divided into a triangle and a rectangle.

Step 2 Find the area of the triangle.



The height, h, of the triangle is 3 cm.

To determine the base length, subtract the two known horizontal lengths: 10 cm − 6 cm = 4.

So, the base, b, of the triangle is 4 cm.

A of triangle $= \frac{1}{2}bh$

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

Step 3 Find the area of the rectangle.

The length, l, is 10 cm, and the width, w, is 9 cm.

A of rectangle $= lw$

$$= 10 \times 9 = 90 \text{ cm}^2$$

Step 4 Add those areas to find the total area of the figure.

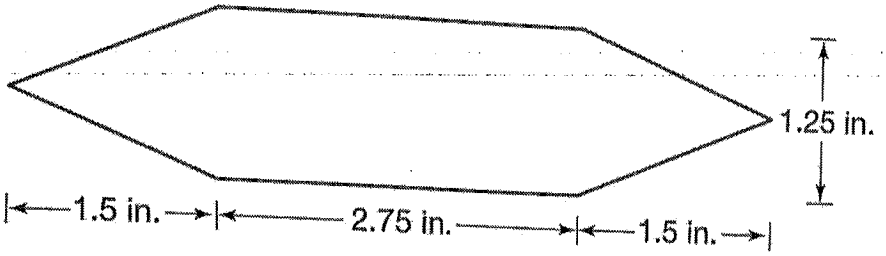
A of composite figure $= 6 + 90 = 96$ cm^2

Solution **The area of the figure is 96 square centimeters.**

Sometimes you will not need an exact answer. Use rounding to find an estimate.

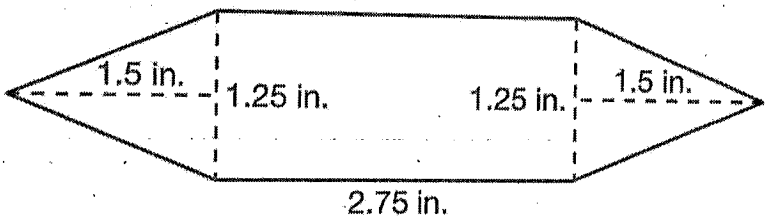
Example 3

What is the approximate area of the figure?



Strategy **Divide the figure into shapes whose area formulas you know.**

Step 1 Divide the figure into 1 rectangle and 2 triangles.



Step 2 Find the approximate area of the rectangle.

$A = lw$

$A = 2.75 \times 1.25$

$\approx 3 \times 1$ Round each decimal to the nearest whole number.

≈ 3 in.^{2}

Step 3 Find the approximate area of the 2 triangles.

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 1.25 \times 1.5$

$\approx \frac{1}{2} \times 1 \times 2$ Round each decimal to the nearest whole number.

≈ 1 in.^{2}

There are 2 triangles: 2×1 in.$^2 = 2$ in.^{2}

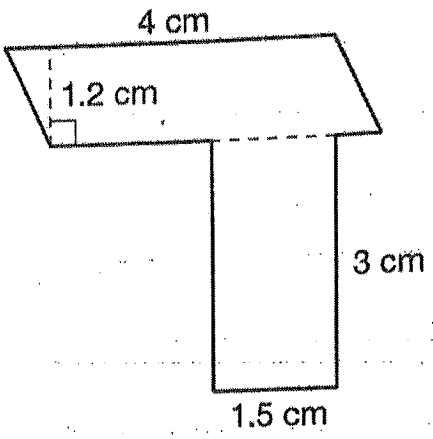
Step 4 Add to find the approximate total area of the figure.

3 in.^{2} + 2 in.^{2} = 5 in.^{2}

Solution **The approximate area of the figure is 5 square inches.**

Coached Example

Aster made a sticker in the shape shown below.



What is the area of the sticker?

A parallelogram and a _____ are combined to form the sticker.

What is the formula for the area of a parallelogram? _____

Find the area of the parallelogram.

What is the formula for the area of a rectangle? _____

Find the area of the rectangle.

Add to find the total area of the figure.

The area of the sticker is _____.

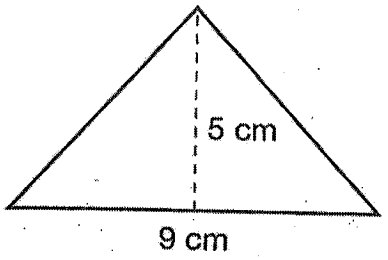
Lesson Practice

Choose the correct answer.

1. A rectangular playground is 85 feet long and 60 feet wide. What is the area of the playground?

   A. 290 ft^2
   B. 510 ft^2
   C. 2,550 ft^2
   D. 5,100 ft^2

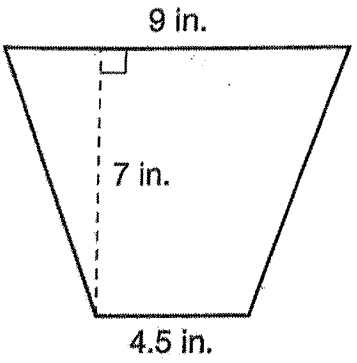
2. A banner is shaped like the triangle shown below.



   What is the area of the banner?

   A. 14 cm^2
   B. 22.5 cm^2
   C. 28 cm^2
   D. 45 cm^2

3. A window is shaped like the trapezoid shown below.



   What is the area of the window?

   A. 47.25 $in.^2$
   B. 63 $in.^2$
   C. 141.75 $in.^2$
   D. 283.5 $in.^2$

4. Which of the figures below have the same area?

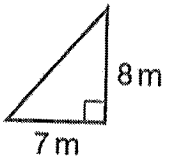


Figure 1

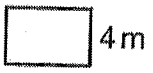


Figure 2

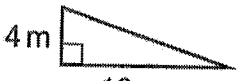


Figure 3

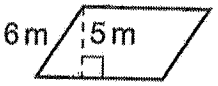


Figure 4

   A. figures 1 and 2
   B. figures 2 and 3
   C. figures 2 and 4
   D. figures 3 and 4

5. What is the area of a parallelogram with a base length of 12 yd and a height of 9 yd?

   A. 60 square yards
   B. 90 square yards
   C. 108 square yards
   D. 120 square yards

6. Which figure has the least area?

   A. square with a side length of 9 cm
   B. parallelogram with a base of 12 cm and a height of 6 cm
   C. triangle with a base of 18 cm and a height of 6 cm
   D. rectangle with a width of 7 cm and a length of 8 cm

Surface Area

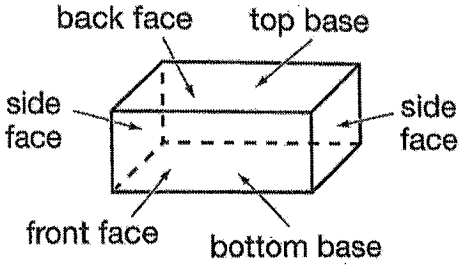
Common Core State Standard:
7.G.6

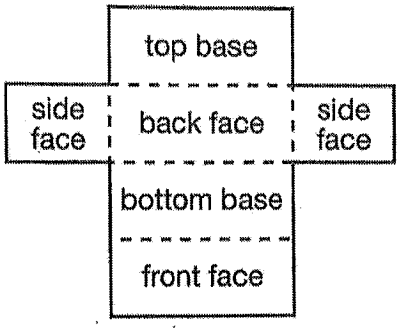
Getting the Idea

The **surface area**, measured in square units, of a solid figure is the sum of the areas of all the surfaces of the figure. You can calculate the surface area of a figure, such as a cube, by finding the areas of all of its **faces** and then adding them.

Looking at a two-dimensional representation, called a **net**, of a solid figure may help you do this.

If the net below is folded along the dotted lines, a rectangular prism is formed.

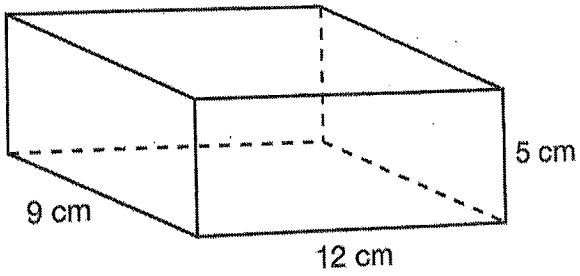




The surface area of the rectangular prism is the total area of the 6 rectangular faces. The formula for the surface area of a rectangular prism is $SA = 2lw + 2lh + 2wh$, where l is length, w is width, and h is height.

Example 1

What is the surface area of this rectangular prism?



Strategy **Use the formula for the surface area of a rectangular prism.**

Substitute the values into the formula and simplify.

Let $l = 12$ cm, $w = 9$ cm, and $h = 5$ cm.

$SA = 2lw + 2lh + 2wh$

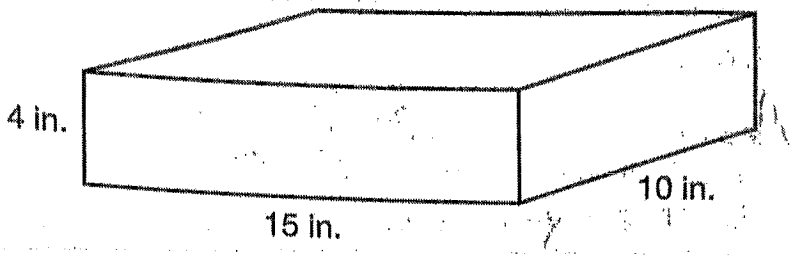
$SA = (2 \times 12 \text{ cm} \times 9 \text{ cm}) + (2 \times 12 \text{ cm} \times 5 \text{ cm}) + (2 \times 9 \text{ cm} \times 5 \text{ cm})$

$SA = 216 \text{ cm}^2 + 120 \text{ cm}^2 + 90 \text{ cm}^2 = 426 \text{ cm}^2$

Solution **The surface area of the rectangular prism is 426 square centimeters.**

Example 2

Mary Jane is going to wrap this box with wrapping paper.



What is the minimum amount of wrapping paper she will need?

Strategy **Find the surface area of a rectangular prism.**

Step 1 Use the formula for the surface area of a rectangular prism.

$SA = 2lw + 2lh + 2wh$

Step 2 Substitute the values for the length, width, and height.

Let $l = 15$ in., $w = 10$ in., and $h = 4$ in.

$SA = 2lw + 2lh + 2wh$

$SA = (2 \times 15 \text{ in.} \times 10 \text{ in.}) + (2 \times 15 \text{ in.} \times 4 \text{ in.}) + (2 \times 10 \text{ in.} \times 4 \text{ in.})$

Step 3 Multiply and add to find the surface area.

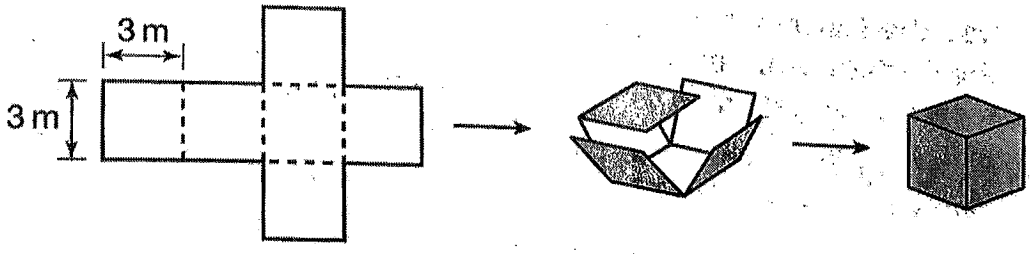
$SA = 300 \text{ in.}^2 + 120 \text{ in.}^2 + 80 \text{ in.}^2$

$SA = 500 \text{ in.}^2$

Solution **The minimum amount of wrapping paper that Mary Jane will need is 500 square inches.**

Example 3

The net below can be folded to form a cube.



What is the surface area of the cube that it will form?

Strategy **Think about the area of a square and the number of [illegible] in a cube.**

Step 1 Find the area of one square in the net.

The leftmost square has dimensions 3 m by 3 m.

A of square face $= s^2 = 3^2 = 9$ square meters

Step 2 Find the total surface area.

The net is made up of 6 congruent squares.

This must be true because a cube has 6 congruent square faces.

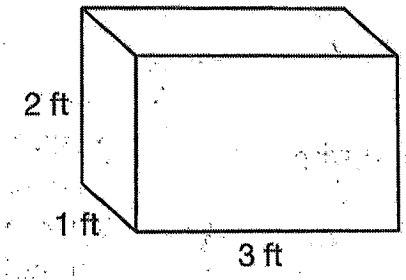
The total surface area is $6 \times$ (area of one face).

$6 \times s^2 = 6 \times 9$ square meters $= 54$ square meters

Solution **The total surface area of the cube is 54 square meters.**

Coached Example

A toymaker will paint four sides of this toy chest. He will not paint the bottom or top surface. How many square feet of the chest will the toymaker paint?



You only need to find the areas of the surfaces that will be painted.

The front and back faces are rectangles that are 3 ft long and _____ ft high.

A of front face $= lw = 3 \times$ _____ $=$ _____ ft^2

The area of the back face is also _____ ft^2.

The left and right side faces are rectangles that are 1 ft long and _____ ft high.

A of left side face $= lw = 1 \times$ _____ $=$ _____

The area of the right side face is also _____ ft^2.

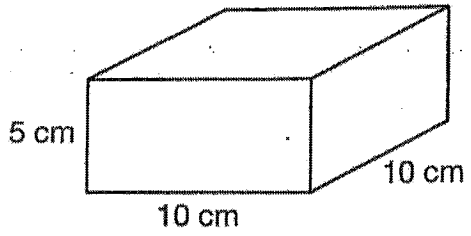
Add the areas of all four faces: _____ + _____ + _____ + _____ = _____

The toymaker will paint _____ square feet of the toy chest.

Lesson Practice

Choose the correct answer.

1. Hermione made a jewelry box shaped like the rectangular prism below. What is the surface area of the jewelry box?

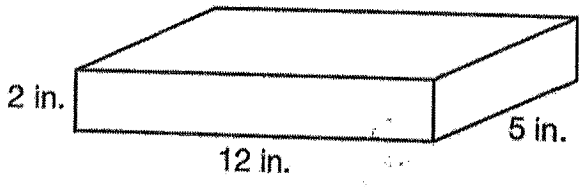


A. 400 cm^2
B. 300 cm^2
C. 200 cm^2
D. 100 cm^2

2. What is the surface area of a cube with 1-inch sides?

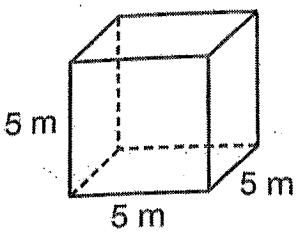
A. 36 $in.^2$
B. 24 $in.^2$
C. 6 $in.^2$
D. 4 $in.^2$

3. What is the surface area of the rectangular prism below?



A. 68 $in.^2$
B. 94 $in.^2$
C. 188 $in.^2$
D. 376 $in.^2$

4. What is the surface area of the cube below?

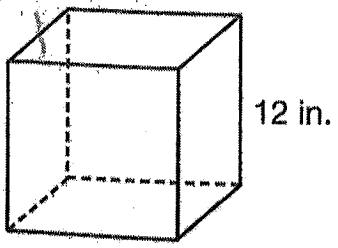


A. 30 m^2
B. 75 m^2
C. 125 m^2
D. 150 m^2

5. Alexis is designing a storage chest shaped like a rectangular prism. The storage chest is 6 feet long, 2 feet wide, and 3 feet high. What is the surface area of the storage chest?

A. 18 ft^2
B. 24 ft^2
C. 36 ft^2
D. 72 ft^2

6. What is the surface area of the cube below?



A. 144 $in.^2$
B. 288 $in.^2$
C. 864 $in.^2$
D. 1,200 $in.^2$

Volume

Getting the Idea

The **volume** of a solid figure is the number of cubic units that fit inside it. You can use the formula below to calculate the volume of any rectangular prism, including a cube.

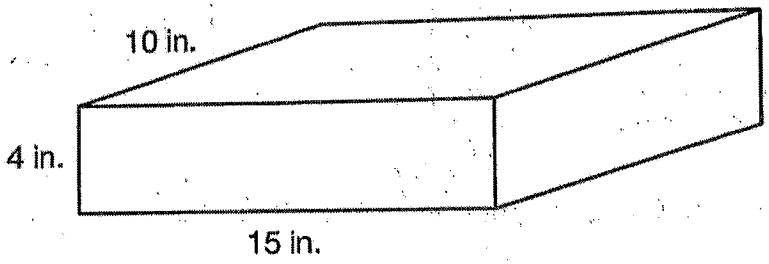
Rectangular Prism

$V = lwh$, where l is the length, w is the width, and h is the height.

Example 1

A department store uses the box below for shirts.

What is the volume of the box?



Strategy **Use the formula for the volume of a rectangular prism.**

Step 1 The box is a rectangular prism. Write the formula for the volume.

$V = lwh$, where l is the length, w is the width, and h is the height.

Step 2 Identify the values for the variables.

The length is 15 in., the width is 10 in., and the height is 4 in.

So $l = 15$ in., $w = 10$ in., and $h = 4$ in.

Step 3 Substitute the values for the variables. Then multiply.

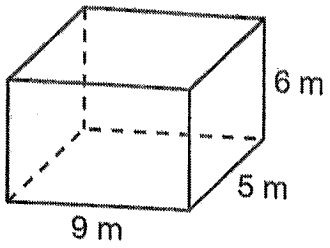
$V = lwh$

$V = 15 \text{ in.} \times 10 \text{ in.} \times 4 \text{ in.} = 600 \text{ in.}^3$

Solution **The volume of the box is 600 cubic inches.**

Example 2

What is the volume of this rectangular prism?



Strategy **Use the formula for the volume of a rectangular prism.**

Step 1 Identify the values for the variables.

The length is 9 m, the width is 5 m, and the height is 6 m.

So $l = 9$ m, $w = 5$ m, and $h = 6$ m.

Step 2 Substitute those values into the formula and solve.

$V = lwh$

$V = 9 \times 5 \times 6$

$V = 270 \text{ m}^3$

Solution **The volume of the rectangular prism is 270 cubic meters.**

Coached Example

Carol has a planter box that is in the shape of a cube. The box measures 20 inches on each side. What is the volume of Carol's planter box?

The formula for the volume of a cube is $V =$ _____.

The length is _____ inches, the width is _____ inches, and the height is _____ inches.

Substitute the values for the variables.

$V =$ _____ $\times$ _____ $\times$ _____

Multiply.

$V =$ _____ in.^3

The planter box has a volume of _____ cubic inches.

Lesson Practice

Choose the correct answer.

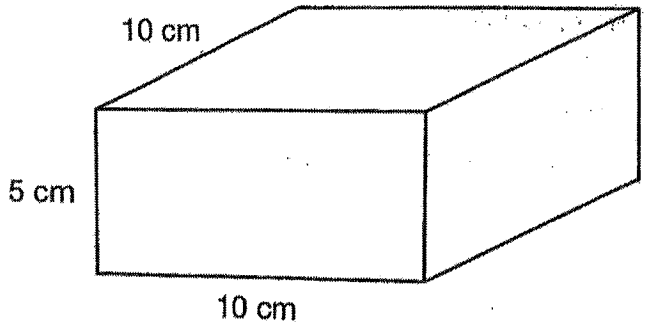
1. A rectangular prism has a length of 3 cm, a width of 7 cm, and a height of 2 cm. What is the volume of the prism?

   A. 21 cm^3
   B. 30 cm^3
   C. 42 cm^3
   D. 45 cm^3

2. A jewelry box is shaped like a cube. The sides of the box are 8 inches long. What is the volume of the jewelry box?

   A. 64 cubic inches
   B. 384 cubic inches
   C. 484 cubic inches
   D. 512 cubic inches

3. What is the volume of this rectangular prism?

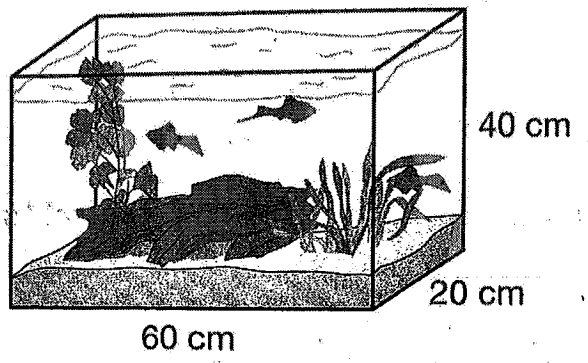


   A. 250 cm^3
   B. 500 cm^3
   C. 750 cm^3
   D. 5,000 cm^3

4. A cube has side lengths of 5 meters. What is the volume of the cube?

   A. 15 m^3
   B. 125 m^3
   C. 150 m^3
   D. 750 m^3

5. Which is the volume of the fish tank below?



   A. 48,000 cm^3
   B. 40,000 cm^3
   C. 36,000 cm^3
   D. 18,000 cm^3

6. A salt shaker is in the shape of a cube and has side lengths of 4 centimeters. What is the volume of the salt shaker?

   A. 8 cm^3
   B. 16 cm^3
   C. 32 cm^3
   D. 64 cm^3

Domain 4: Cumulative Assessment for Lessons 19–27

1. A pole that is 12 feet tall casts a shadow that is 9 feet long. At the same time of day, how long is the shadow cast by a tree that is 45 feet tall?

   A. 27 ft
   B. 29.25 ft
   C. 31.5 ft
   D. 33.75 ft

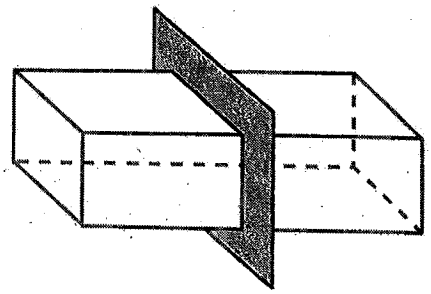
2. The scale on a map is 2 inches = 75 miles. If two cities are 575 miles apart, how many inches apart are they on the map?

   A. $3\frac{5}{6}$ in.
   B. $7\frac{2}{3}$ in.
   C. $11\frac{1}{2}$ in.
   D. $15\frac{1}{3}$ in.

3. Carson wants to construct a triangle with side lengths of 4 inches, 7 inches, and 12 inches. Which best describes a triangle with those side lengths?

   A. a unique, right triangle
   B. a unique, acute triangle
   C. nonexistent
   D. ambiguously defined

4. A rectangular prism is sliced by a plane that is perpendicular to its base, as shown. What is the shape of the cross section formed?

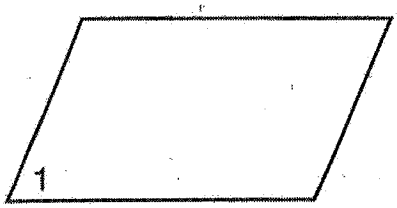


   A. trapezoid
   B. square
   C. rectangle (not a square)
   D. parallelogram (not a rectangle)

5. A circular clock has a radius of 8.5 centimeters. Which is closest to the circumference of the clock?

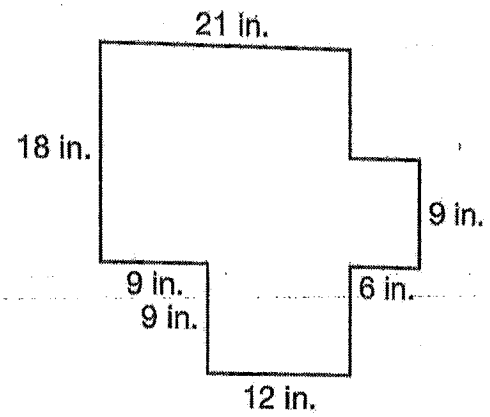
A. 14.13 cm
B. 26.69 cm
C. 53.38 cm
D. 226.87 cm

6. If m∠1 is 55° in the parallelogram below, what is the measure of its supplementary angle?



A. 35°
B. 45°
C. 115°
D. 125°

7. Brook made a base to set under her ferret's cage. The drawing below shows the shape of the base.



What is the area of the base?

A. 486 $in.^2$
B. 540 $in.^2$
C. 567 $in.^2$
D. 729 $in.^2$

8. A rectangular prism has length 9 centimeters, width 6 centimeters, and height 8 centimeters. What is the volume of the rectangular prism?

A. 432 cm^3
B. 328 cm^3
C. 216 cm^3
D. 164 cm^3

Probability

Common Core State Standards:
7.SP.5, 7.SP.6, 7.SP.7.a, 7.SP.7.b

Getting the Idea

Probability measures the chance of an event happening based on the number of the **possible outcomes.** Probability can be expressed as a fraction or a decimal from 0 to 1. A probability close to 0 means an event is unlikely. A probability close to 1 means an event is very likely. A probability close to $\frac{1}{2}$ or 0.5 means an event is neither unlikely nor likely. You can also express a probability as a percent.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur **(favorable outcome)** to the number of possible outcomes. The probability, P, of an event, A, is:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

Example 1

Josh is going to choose a random card from 13 cards. The cards are numbered from 1 to 13. What is the probability that he will choose a card with a number less than 5? Determine if the event is likely, unlikely, or neither.

Strategy **Find the theoretical probability.**

Step 1 Count the number of favorable outcomes.

There are 4 cards (1, 2, 3, 4) with a number less than 5.

Step 2 Count the number of possible outcomes.

There are a total of 13 cards, each with the same chance of being drawn.

Step 3 Find the theoretical probability.

$$P(\text{card with a number less than 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{13}$$

Step 4 Determine if the event is likely or unlikely.

$\frac{4}{13}$ is closer to 0 than it is to 1, and it is less than $\frac{1}{2}$. So, the event is unlikely.

Solution **The probability of choosing a card with a number less than 5 is $\frac{4}{13}$. The event is unlikely.**

You can use theoretical probability to make a prediction. Multiply the theoretical probability by the number of **trials**, or times the experiment is performed to predict the number of favorable outcomes.

Example 2

Peter will roll a number cube, labeled 1 through 6, a total of 90 times. What is a good prediction for the number of times that the number cube will land on 5?

Strategy **Find the number of possible outcomes and favorable outcomes.**

Step 1 Find the number of possible outcomes.

There are 6 possible outcomes for the number cube.

Step 2 Find the number of favorable outcomes.

There is one 5 on the number cube.

Step 3 Write the theoretical probability in simplest form.

$$P(\text{rolling a 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 4 Multiply the probability by the number of trials.

$$\frac{1}{6} \times 90 = 90 \div 6 = 15$$

Solution **A good prediction is that Peter will roll a 5 about fifteen times.**

Experimental probability is the ratio of the total number of times the favorable outcome happens to the total number of trials, or times the experiment is performed. The experimental probability, P_e, of event A is:

$$P_e(A) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}.$$

Experimental probability is useful when you need to make predictions about an event. As the number of trials increases, the experimental probability gets closer to the theoretical probability.

Example 3

Minnie conducted an experiment with a spinner. The results are shown in the table.

Number	1	2	3	4	5	6
Times Landed	10	7	6	8	5	6

Based on the data, what is the probability that the spinner will land on 2 on the next spin?

Strategy **Find the experimental probability.**

Lesson Practice

Choose the correct answer.

Use the following event for questions 1 and 2.

Dan rolled a number cube 20 times. The cube landed on the number 3 six times.

1. What is the experimental probability that Dan will roll a number 3 the next time he rolls the number cube?

   A. $\frac{3}{10}$

   B. $\frac{2}{5}$

   C. $\frac{3}{5}$

   D. $\frac{7}{10}$

2. Which best describes what would likely happen if Dan rolled the number cube another 80 times?

   A. There would be no change.

   B. The experimental probability would get farther from the theoretical probability.

   C. The experimental probability would exactly match the theoretical probability.

   D. The experimental probability would get closer to the theoretical probability.

3. Sonya wrote each letter of LEDBETTER on a separate index card and put the cards in a box. She picked one letter at random, put the card back, and then repeated the experiment. If she performed this experiment 90 times, which is the best prediction for the number of times that Sonya would pick a T?

   A. 10

   B. 20

   C. 25

   D. 30

4. A lightbulb manufacturer found that out of 200 lightbulbs, 15 were defective. How many lightbulbs should the manufacturer expect to be defective out of 2,400 lightbulbs?

   A. 40

   B. 80

   C. 120

   D. 180

Compound Events

Common Core State Standards:
7.SP.8.a, 7.SP.8.b, 7.SP.8.c

Getting the Idea

A **compound event** is a combination of two or more events. Compound events can be dependent or independent. Events are **independent** when the outcome of one event does not affect the outcome of a second event. When the outcome of one event affects the outcome of a second event, the events are **dependent.**

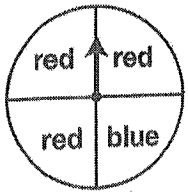
As with simple events, the probability of a compound event is the ratio of favorable outcomes to total outcomes in the sample space for which the compound events occur. You can use tables, organized lists, and tree diagrams to find the probability of compound events, or you can use the rules below.

To find the probability of two independent events, multiply the probability of the first event by the probability of the second event.

$P(\text{two independent events}) = P(\text{first event}) \times P(\text{second event})$

Example 1

Adriana tosses a number cube with faces numbered 1 through 6 and spins the spinner shown below at the same time.



What is the probability of tossing a number greater than 2 on the cube and spinning red on the spinner? Express the probability as a fraction, as a percent, and as a decimal.

Strategy **Find the probability of each event and multiply them together.**

Step 1 Decide if the events are dependent or independent.

The outcome on the number cube does not affect the outcome on the spinner.

The events are independent.

Step 2 Find the probability of the number cube landing on a number greater than 2.

A number cube has 6 possible outcomes.

Four outcomes (3, 4, 5, 6) are greater than 2.

$$P(>2) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{6} = \frac{2}{3}$$

Step 3 Find the probability of spinning red on the spinner.

Three of the 4 sections are labeled "red."

$$P(\text{red}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{4}$$

Step 4 Multiply the two probabilities.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

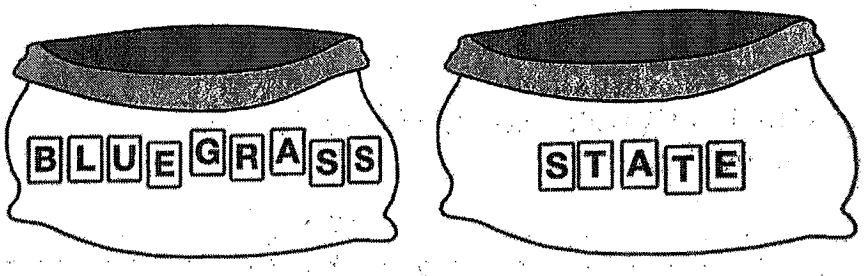
Step 5 Express the probability as a fraction, decimal, and percent.

$$\frac{1}{2} = 0.5 = 50\%$$

Solution **The probability of the cube landing on a number greater than 2 and the spinner landing on red is $\frac{1}{2}$, 0.5, or 50%.**

Example 2

Dion wrote the letters of the Kentucky state nickname on a set of same-sized cards and placed the cards into two bags as shown below.



He will choose one card from each bag without looking. What is the probability that he will choose the letter A from each bag?

Strategy **Find the probability of each event and multiply.**

Step 1 Decide if the events are dependent or independent.

The letter drawn from the first bag does not affect the letter drawn from the second bag.

The events are independent.

Step 2 Find the probability of choosing an A from the first bag.

One of the 9 letters is an A.

$P(A)$ for first bag $= \frac{1}{9}$

Step 3 Find the probability of choosing an A from the second bag.

One of the 5 letters is an A.

$P(A)$ for second bag $= \frac{1}{5}$

Step 4 Multiply the probabilities.

$\frac{1}{9} \times \frac{1}{5} = \frac{1}{45}$

Solution **The probability that Dion will choose an A from each bag is $\frac{1}{45}$.**

When you need to find the probability of a compound event, sometimes it is necessary to make a tree diagram, an organized list, or a table to find the number of possible outcomes.

You can also use the **fundamental counting principle** to find the number of possible outcomes. If event A can occur in m ways and event B can occur in n ways, then events A and B can occur in $m \times n$ ways.

Example 3

What is the probability of tossing a sum of 9 on two number cubes, each numbered 1 through 6?

Strategy **Find the number of possible outcomes. Then make an organized list.**

Step 1 Use the fundamental counting principle to find the number of possible outcomes.

There are 6 possible outcomes for each number cube.

$6 \times 6 = 36$, so there are 36 possible outcomes.

Step 2 Make an organized list of all of the ways to get a sum of 9 from the number cubes.

$3 + 6 \quad 4 + 5 \quad 5 + 4 \quad 6 + 3$

There are 4 ways to get a sum of 9.

Step 3 Write the probability in simplest form.

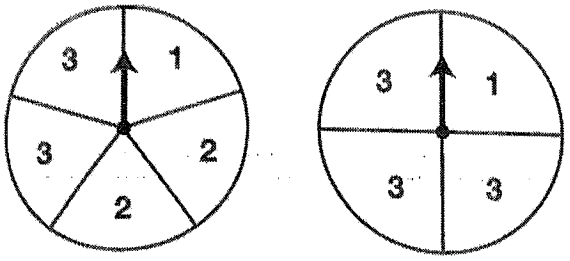
$\frac{4}{36} = \frac{1}{9}$

Solution **The probability of tossing a sum of 9 on two number cubes is $\frac{1}{9}$.**

Lesson Practice

Choose the correct answer.

Use the spinners below for questions 1 and 2.



1. What is the probability of spinning a 3 on both of the spinners?

   A. 0.25
   B. 0.3
   C. 0.35
   D. 0.4

2. What is the probability of spinning a sum of 4 when spinning both spinners at the same time?

   A. 20%
   B. 25%
   C. 30%
   D. 35%

3. Gabriel is doing a probability experiment. He is tossing a coin and spinning a spinner with 4 equal sections numbered from 1 through 4. How many possible outcomes are there?

   A. 2
   B. 4
   C. 6
   D. 8

4. There are 4 boys and 2 girls from the seventh grade and 3 boys and 5 girls from the eighth grade on the soccer team. Coach Hart will pick one captain from each grade. What is the probability that both captains will be girls?

   A. $\frac{5}{24}$
   B. $\frac{1}{4}$
   C. $\frac{3}{10}$
   D. $\frac{1}{2}$

Samples

Common Core State Standards:
7.SP.1, 7.SP.2

Getting the Idea

Statistics can be used to make generalizations about a population. A **population** is the group of interest. It is usually not possible to gather data from each member of a population, so the generalizations are often based upon a sample. A **sample** is a smaller group taken from the population.

Samples allow researchers to save time and money when gathering information. Samples are only useful if they are representative of the population. A **representative sample** is a portion of the population that is similar to the entire population. A **biased sample** is one in which some members of the population have a greater chance of being selected for the sample than other members. Because of bias, the sample does not fairly represent the population.

One way to gather information is by surveying the members of the sample. A **survey** is a question or set of questions used to gather **data**, or pieces of information. A survey can also be biased.

Example 1

Reggie thinks that more students in his school are right-handed than left-handed. He surveys the students in his class and finds that 23 of the 27 students are right-handed. Do the results of Reggie's survey support his inference that more students in his school are right-handed than left-handed?

Strategy **Use the definition of a representative sample to evaluate Reggie's sample.**

A sample should be representative of the population.

The population being studied is the students in Reggie's school.

The students in Reggie's class are representative of all students in his school.

More students in his class are right-handed than left-handed.

The results of the survey support his inference.

Solution **The results of Reggie's survey support his inference that more students in his school are right-handed than left-handed.**

Random samples are usually preferred when gathering information about a population. In a random sample, each individual in the population has an equal chance of being part of the sample.

Example 2

Collin asked every eighth student entering the school which of four subjects was his or her favorite. Can the results of Collin's survey be used to draw inferences about students' favorite subjects at the school?

Strategy **Decide if the sample is representative and the survey is unbiased.**

Step 1 Decide if the sample is a random sample.

In a random sample, each individual in the population has the same chance of being part of the sample. Each student entering the school has the same chance of being one of every eight students entering the school.

Step 2 Decide if the sample is representative of the population.

The students in the school are the population.

The random sample is representative of the school population.

Step 3 Decide if the survey is biased.

The results are only representative of the four subjects included in the survey. It is biased toward these four subjects since other subjects are not included. The results can only be used to draw inferences about student preferences for the four subjects included in the survey.

Solution **The results of Collin's survey can be used to draw inferences about student preferences for the four subjects included in the survey.**

You can use the results of a survey to make predictions about a population.

Example 3

The table below shows the results of Collin's survey from Example 2.

Favorite Subject

Subject	Number of Students
Math	15
Science	20
Language Arts	10
Social Studies	5

There are 400 students at Collin's school. How many students would you predict prefer language arts?

Strategy **Write and solve a proportion.**

Step 1 Add to find the total number of students Collin surveyed.

$15 + 20 + 10 + 5 = 50$

Lesson Practice

Choose the correct answer.

1. Martha is planning to survey people at a water park to determine the most popular water slide at the park. Which would be the best sample for her survey to draw a valid inference?

   A. children at the park between the ages of 3 and 5

   B. children at the park between the ages of 6 and 10

   C. adults at the park between the ages of 20 and 30

   D. adults and children of all ages at the park

2. A newspaper is conducting a survey to determine which American professional baseball team is most popular. How would it most likely get a random sample that is representative of the population?

   A. by asking people at a Florida Marlins game

   B. by calling people from around the country

   C. by asking every fifth person entering the stadium at a Red Sox game

   D. by asking people at a Cincinnati Reds game

Use the information below for questions 3 and 4.

Mr. Callahan just opened a flower shop. He took a random survey of shoppers to find out their favorite flowers and recorded the results in the table below.

Favorite Flower

Type	Shoppers
Daffodil	14
Lily	10
Rose	24
Daisy	12

3. What is the size of the sample?

   A. 4

   B. 50

   C. 60

   D. 64

4. If Mr. Callahan expects to sell 150 bunches of flowers next week, which is the best prediction of how many bunches of daffodils he should have in his shop?

   A. 28

   B. 35

   C. 42

   D. 60

Measures of Central Tendency

Getting the Idea

Measures of central tendency help to describe and interpret a data set. They are used to interpret the "average item" of a data set. The table below shows the measures of central tendency for this data set: 5, 1, 1, 6, and 7.

Measure of Central Tendency	Example
The **mean** is equal to the sum of the terms in a data set divided by the number of terms in the data set.	mean $= \frac{\text{sum of terms}}{\text{number of terms}}$ mean $= \frac{5+1+1+6+7}{5} = \frac{20}{5} = 4$
The **median** is the middle term in a data set ordered from least to greatest. If there is an even number of terms in a data set, the median is the mean of the two middle numbers.	The data ordered from least to greatest are: 1, 1, 5, 6, 7. The middle term, 5, is the median.
The **mode** is the term or terms that appear most often in a data set. A data set may have no mode, one mode, or more than one mode.	5, 1, 1, 6, 7 The number 1 appears twice in the data set. Every other number appears only once. So, 1 is the mode.

Example 1

The scores on a science quiz are: 7, 7, 9, 7, 10, 8, 6, 9, 10, and 7.

What are the mean, median, and mode of the science quiz scores?

Strategy **Calculate each measure of central tendency.**

Step 1 Find the mean.

Add the values and then divide the sum by the number of values.

$7 + 7 + 9 + 7 + 10 + 8 + 6 + 9 + 10 + 7 = 80$

$80 \div 10 = 8$

Step 2 Find the median.

Order the values from least to greatest.

6, 7, 7, 7, 7, 8, 9, 9, 10, 10

There is an even number of terms.

The two middle values are 7 and 8.

Find the mean of the two middle values.

$(7 + 8) \div 2 = 15 \div 2 = 7.5$

Step 3 Find the mode.

The value that occurs most frequently is 7.

Solution **The mean is 8, the median is 7.5, and the mode is 7.**

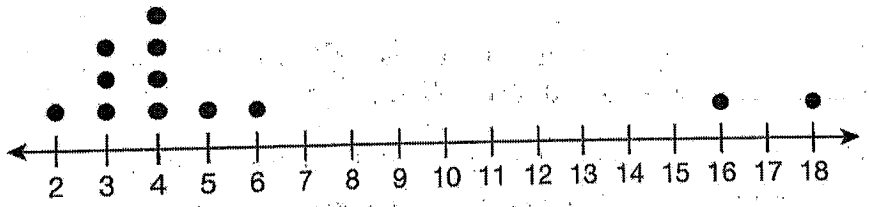
To choose which measure of central tendency is most appropriate for a situation, look at the distribution of the ordered data from a sample. If there is a value that is much less or much greater than the other values in the set of data, the median or mode better represents the sample data than the mean. If the mode occurs at either extreme of the data, the mean or the median are better choices to represent the sample data.

Some data sets are shown on a **dot plot**, which is a display that uses a number line and dots to show data.

Example 2

Karen randomly surveyed some classmates to see how many books each of them read over the summer. The results of her survey are shown in the dot plot.

Number of Books Read



How does the mode compare to the median number of books read?

Strategy **Find the median and the mode. Compare the measures.**

Step 1 Find the median.

Use the data points shown on the dot plot.

Median: 2, 3, 3, 3, 4, **4**, **4**, 4, 5, 6, 16, 18

The two middle values are 4.

The median is 4.

Step 2 Find the mode.

Mode: 2, 3, 3, 3, **4**, **4**, **4**, **4**, 5, 6, 16, 18

The mode is 4.

Step 3 Compare the measures.

The median and the mode are the same.

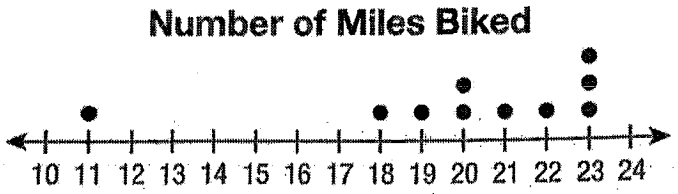
Solution **The median and the mode are both 4.**

Lesson Practice

Choose the correct answer.

Use the dot plot for questions 1–3.

The dot plot shows the number of miles Jamal biked per week for ten weeks.



1. What is the mean number of miles that Jamal bikes per week?

   A. 20 miles
   B. 20.5 miles
   C. 21 miles
   D. 23 miles

2. What is the median number of miles that Jamal bikes per week?

   A. 19 miles
   B. 20 miles
   C. 20.5 miles
   D. 21 miles

3. Which measure of central tendency best represents the average number of miles that Jamal bikes per week?

   A. mean or mode
   B. mean or median
   C. median or mode
   D. mean, median, or mode

Use the tables for questions 4–6.

The tables show the quiz scores of students in two seventh grade social studies classes.

Quiz Scores

Class A

9	8	8	9
10	9	8	10
9	9	10	

Class B

9	10	8	10
10	9	10	6
5	7	8	10

4. Which best describes the comparison between the mode quiz scores?

   A. The modes are the same.
   B. The mode score for Class A is 2 points higher than for Class B.
   C. The mode score for Class A is 1 point higher than for Class B.
   D. The mode score for Class A is 1 point lower than for Class B.

5. Which best describes the comparison between the mean quiz scores?

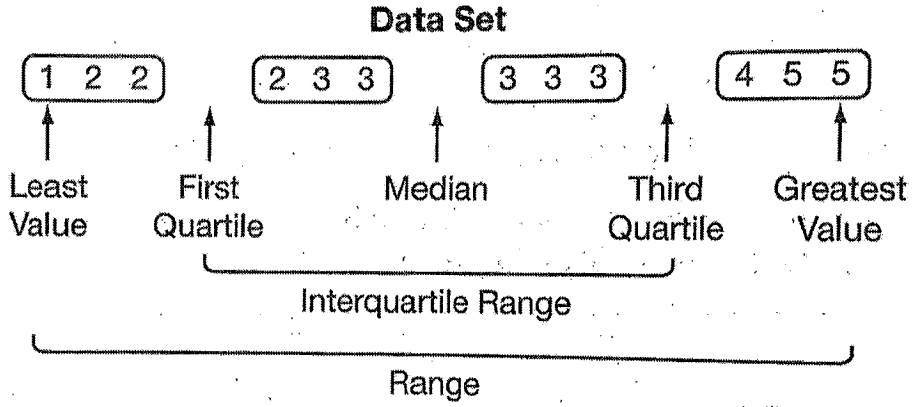
   A. The means are the same.
   B. The mean score for Class A is 0.5 point higher than for Class B.
   C. The mean score for Class A is 1 p higher than for Class B.
   D. The mean score for Class A is 1 p lower than for Class B.

Measures of Variation

Getting the Idea

Instead of describing the center of a set of data by using the measures of central tendency, you may wish to describe the spread of a set of data. Measures of variation show how spread out or close together the data in a set are, or how much the data points vary.

When data are arranged from least to greatest, the median divides the data into two equal halves. The **first quartile** is the median of the data values that are less than the median. The **third quartile** is the median of the data values that are greater than the median. The **quartiles** and the median divide the data into four quarters. The **range** is the difference between the greatest value in a data set and the least value. The **interquartile range (IQR)**, is the difference between the third quartile and the first quartile. The range measures the spread of all the data. The IQR measures the spread of the two middle quarters of the data.



For the data set in the diagram, the median is 3. The first quartile is 2 (the mean of 2 and 2). The third quartile is 3.5 (the mean of 3 and 4). The range is 5 − 1, or 4. The IQR is 3.5 − 2, or 1.5.

In a data set, a number that is much less or much greater than the other numbers in the data set is an **outlier**. A data set may contain one or more outliers. An outlier will affect the range, but it will not affect the interquartile range. This is why the median is not as affected by outliers as the mean is.

Consider the following data set: 2, 3, 5, 6, 8, 10, 23.

The median is 6.

The first quartile is 3, which is the median of the values 2, 3, 5.

The third quartile is 10, which is the median of the values 8, 10, 23.

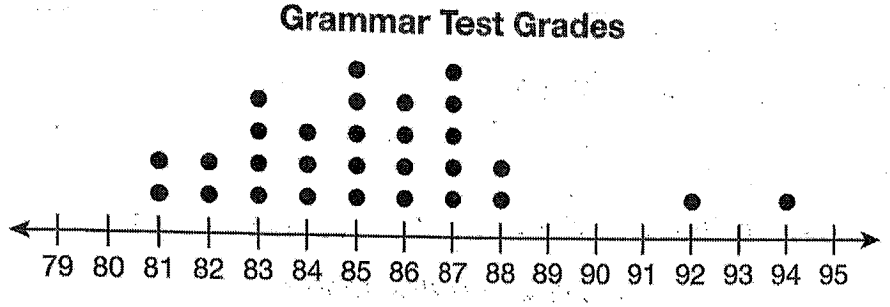
The range is 21: 23 − 2 = 21.

The IQR is 7: 10 − 3 = 7.

For this data set, 23 is an outlier. It is much greater than the other numbers in the set. The range is affected by this outlier. It suggests that there is a greater variability in the data, since it shows a greater spread, than the IQR suggests. The data vary by 21 from the least to greatest values, while they only vary by 7 away from the median, or the center of the data.

Example 1

The dot plot below shows the grades students received on a grammar test in Ms. Parsi's class.



What is the range in the grades? What is the median grade?

Strategy **Use the dot plot.**

Step 1 Find the greatest and least test grades.

The lowest grade is 81.

The highest grade is 94.

Step 2 Find the range.

Subtract the lowest grade from the highest grade.

range = 94 − 81 = 13

Step 3 Count the total number of test grades.

There are 29 grades.

Step 4 Find the median.

The median is the middle grade. It is halfway between the lowest and the highest grade.

Since there are 29 grades, there are 14 grades below the median and 14 grades above the median.

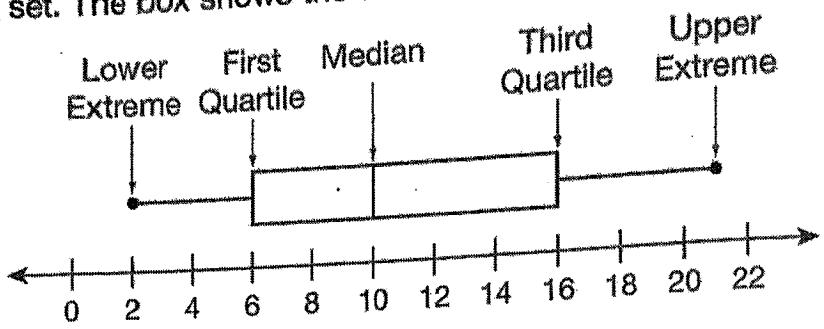
The median is the fifteenth grade in the ordered list of grades.

Start at 81 and count the dots until you reach the fifteenth dot.

This dot is at the grade of 85. So the median is 85.

Solution **The range in the grades is 13, and the median is 85.**

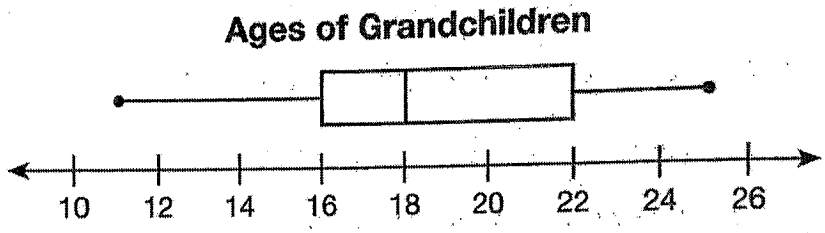
A **box plot**, also known as a box-and-whiskers plot, is a method of visually displaying a distribution of data values by using the median, quartiles, and extremes (least and greatest values) of the data set. The box shows the middle 50% of the data.



A box plot is a good way to show the spread of a set of data.

Example 2

The box plot below shows the ages of Mr. Morehouse's grandchildren.



Find the median, first quartile, third quartile, and interquartile range of their ages.

Strategy **Use the box plot.**

Step 1 Find the median.

The median is the middle value. On a box plot, it is represented by the vertical line inside the box.

The vertical line inside the box is above 18. The median is 18.

Step 2 Find the first quartile.

On a box plot, the first quartile is represented by the box's left vertical line.

The left vertical line is above 16. The first quartile is 16.

Step 3 Find the third quartile.

On a box plot, the third quartile is represented by the box's right vertical line.

The right vertical line is above 22. The third quartile is 22.

Step 4 Find the interquartile range.

$\text{IQR} = \text{third quartile} - \text{first quartile} = 22 - 16 = 6$

Solution **The median of the ages is 18. The first quartile is 16. The third quartile is 22. The interquartile range is 6.**

Example 3

Below are the quiz scores from students in two different class sections.

Section 1: 7, 9, 9, 10, 8, 6, 8, 5, 5, 9, 10, 7, 8, 7, 9

Section 2: 7, 8, 9, 9, 8, 8, 7, 9, 9, 10, 8, 8, 7, 10, 8

Which section has greater variability in the scores?

Strategy **Compare the ranges and interquartile ranges of the two class sections.**

Step 1 Find the range for Section 1.

Order the scores from least to greatest.

5, 5, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10

The highest score was 10. The lowest score was 5.

Range $= 10 - 5 = 5$

Step 2 Find the median and the quartiles for Section 1.

5, 5, 6, $\underline{7}$, 7, 7, 8, $\underline{\underline{8}}$, 8, 9, 9, $\underline{9}$, 9, 10, 10

The median is 8, the first quartile is 7, and the third quartile is 9.

Step 3 Find the interquartile range for Section 1.

The interquartile range (IQR) is the difference between the third and first quartiles.

IQR $= 9 - 7 = 2$

Step 4 Find the range for Section 2.

Order the scores from least to greatest.

7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10

Range $= 10 - 7 = 3$

Step 5 Find the median and the quartiles for Section 2.

7, 7, 7, $\underline{8}$, 8, 8, 8, $\underline{\underline{8}}$, 8, 9, 9, $\underline{9}$, 9, 10, 10

The median is 8, the first quartile is 8, and the third quartile is 9.

Step 6 Find the interquartile range for Section 2.

IQR $= 9 - 8 = 1$

Step 7 Compare the range and interquartile range for each section.

The range for Section 1 is 5, and the IQR is 2.

The range for Section 2 is 3, and the IQR is 1.

The range and IQR for Section 1 are greater than for Section 2.

Solution **Section 1 has greater variability in the quiz scores than Section 2.**

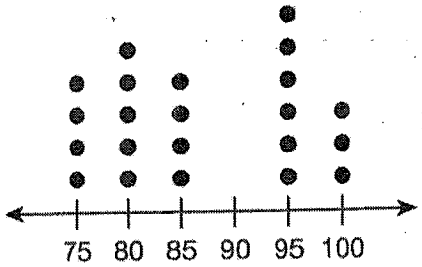
Lesson Practice

Choose the correct answer.

Use the dot plot below for questions 1 and 2.

The dot plot below shows the grades that a class of students received on their recent social studies homework assignment.

Homework Assignment Grades



1. What is the first quartile grade?

   A. 75
   B. 80
   C. 85
   D. 90

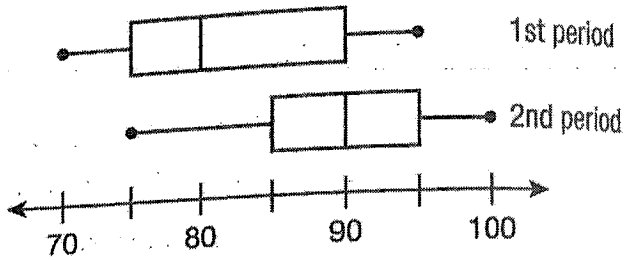
2. What is the third quartile grade?

   A. 85
   B. 90
   C. 95
   D. 100

Use the double box-and-whisker plot for questions 3 and 4.

The double box-and-whisker plot below shows the vocabulary quiz scores for Mr. Edelman's first and second period classes.

Quiz Scores



3. What is the interquartile range of the first period quiz scores?

   A. 5
   B. 10
   C. 15
   D. 20

4. Which statement about the quiz scores is true?

   A. The range of the scores was the same for both classes.
   B. The interquartile range of the scores was the same for both classes.
   C. The mean score was the same for both classes.
   D. About 25% of the students in both classes scored 95 or higher on the quiz.

Mean Absolute Deviation

Getting the Idea

Another way to measure the variability of a data set is to measure variation from the mean. You do this by measuring how far each individual value is from the mean.

To measure variability away from the mean, first find the mean of the data set. Next, find the absolute value of the difference between the mean and each value of the data set. This gives the deviation of each value from the mean. Then find the sum of all the deviations and divide the sum by the number of values in the data set. The average of the absolute deviations from the mean is called the **mean absolute deviation**.

Suppose the heights, in inches, of three plants, are: 18, 27, and 21.

Find the mean of the heights, in inches.

$$\text{mean} = \frac{18 + 27 + 21}{3} = \frac{66}{3} = 22$$

Find how each height differs from the mean height. This is the deviation from the mean.

deviation of first value $= 18 - 22 = -4$

deviation of second value $= 27 - 22 = 5$

deviation of third value $= 21 - 22 = -1$

Notice that the average deviation of the values will be zero.

So, take the absolute value of each deviation.

$|-4| = 4$

$|5| = 5$

$|-1| = 1$

Now find the average of the absolute deviations.

$$\text{mean absolute deviation} = \frac{4 + 5 + 1}{3} = \frac{10}{3} = 3.\overline{3}$$

So, the plant heights vary by an average of $3.\overline{3}$ inches from the mean.

When the mean absolute deviation is small, it means the data is bunched closely together. For the plant heights, the mean absolute deviation is $3.\overline{3}$, which is relatively small. This makes sense since the plant heights are not very different. So, there is not much variability in the plant heights.

If the mean absolute deviation is large, it means the data is spread out and has greater variability.

Example 1

Find the mean absolute deviation for the following quiz scores: 6, 9, 6, 9, 8, and 10. The mean score on the quizzes is 8.

Strategy **Find the deviation of each score from the mean score. Then find the absolute deviations to get the mean absolute deviation.**

Step 1 Find the deviation of each score from the mean score.

Subtract each score from the mean score to find the deviations.

$6 - 8 = -2$

$9 - 8 = 1$

$6 - 8 = -2$

$9 - 8 = 1$

$8 - 8 = 0$

$10 - 8 = 2$

Step 2 Find the absolute deviations, the absolute value of each deviation.

$|-2| = 2$

$|1| = 1$

$|-2| = 2$

$|1| = 1$

$|0| = 0$

$|2| = 2$

Step 3 Find the mean absolute deviation.

Add the absolute deviations: $2 + 1 + 2 + 1 + 0 + 2 = 8$

There are 6 quiz scores in the set.

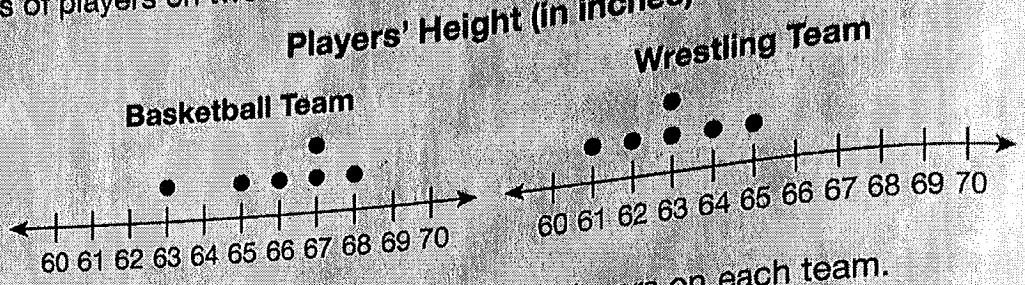
mean absolute deviation $= \frac{8}{6} = 1.\overline{3}$

Solution **The mean absolute deviation is $1.\overline{3}$.**

You can use the mean absolute deviation to compare two populations.

Example 2

The heights of players on two school teams are shown on the dot plots below.



Compare the variability in the mean heights of the players on each team.

Strategy **Find the mean absolute deviation of each team's heights.**

Step 1 Find the mean absolute deviation of the basketball team's heights.

$$\text{mean} = \frac{63 + 65 + 66 + 67 + 67 + 68}{6} = \frac{396}{6} = 66$$

Find the absolute deviation of each height from the mean height.

$|63 - 66| = |-3| = 3$ $\quad$ $|67 - 66| = |1| = 1$

$|65 - 66| = |-1| = 1$ $\quad$ $|67 - 66| = |1| = 1$

$|66 - 66| = |0| = 0$ $\quad$ $|68 - 66| = |2| = 2$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{3 + 1 + 0 + 1 + 1 + 2}{6} = \frac{8}{6} = 1.\overline{3}$$

Step 2 Find the mean absolute deviation of the wrestling team's heights.

$$\text{mean} = \frac{61 + 62 + 63 + 63 + 64 + 65}{6} = \frac{378}{6} = 63$$

Find the absolute deviation of each height from the mean height.

$|61 - 63| = |-2| = 2$ $\quad$ $|63 - 63| = |0| = 0$

$|62 - 63| = |-1| = 1$ $\quad$ $|64 - 63| = |1| = 1$

$|63 - 63| = |0| = 0$ $\quad$ $|65 - 63| = |2| = 2$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{2 + 1 + 0 + 0 + 1 + 2}{6} = \frac{6}{6} = 1$$

Step 3 Compare the mean absolute deviations.

$1.\overline{3} > 1$

The mean absolute deviation of the basketball team's heights is slightly more than the mean absolute deviation of the wrestling team's heights. This corresponds to the dot plots. The dot plot for the basketball team shows somewhat more variability than the dot plot for the wrestling team.

Solution **There is a little more variability in the heights of the basketball team than in the heights of the wrestling team.**

Lesson Practice

Choose the correct answer.

Use the following information for questions 1 and 2.

Paula's grades on her history tests this semester are 79, 93, 92, 86, and 90.

1. Which shows the deviation of each of her grades from her mean grade?

   A. $-9, 5, 4, -2, 2$
   B. $-8, 6, 3, -3, 2$
   C. $-11, 6, 5, -4, 4$
   D. $-14, 9, 5, -3, 2$

2. What is the mean absolute deviation of Paula's history grades?

   A. 0
   B. 4.2
   C. 4.4
   D. 22

Use the following information for questions 3 through 5.

The lengths, in seconds, of four folk songs are 128, 165, 182, and 141.

The lengths, in seconds, of four pop songs are 90, 98, 102, and 94.

3. What is the mean absolute deviation, in seconds, of the folk songs?

   A. 18
   B. 18.25
   C. 19.5
   D. 19.75

4. What is the mean absolute deviation, in seconds, of the pop songs?

   A. 2
   B. 4
   C. 6
   D. 8

5. Which of the following statements is true?

   A. The variability in the times of the folk songs is about half that of the pop songs.
   B. The variability in the times of the folk songs is about twice that of the pop songs.
   C. The variability in the times of the folk songs is about 3 times that of the pop songs.
   D. The variability in the times of the folk songs is about 4 times that of the pop songs.

Make Predictions Using Data

Common Core State Standards:
7.SP.2, 7.SP.4

Getting the Idea

You can use the data from a sample to make predictions about the population. It is important that the sample be representative of the population for the predictions to be reasonable.

Example 1

There are 60 students who take band classes at Mr. Tempo's school. Mr. Tempo surveyed 10 of those students to find out how long they practice their instruments each day. The survey was randomly distributed and anonymous. The results of the survey are shown below. The times are in minutes.

40, 25, 30, 40, 20, 15, 25, 30, 20, 25

Find the mean practice time for the sample. Predict the mean practice time of all the students who take band classes. Is the prediction reasonable?

Strategy **Use the mean from the sample data to predict the mean for the population.**

Step 1 Identify the sample and the population.

The students surveyed are the sample.

The population is all the students who take band classes.

Step 2 Find the mean for the sample data.

$40 + 25 + 30 + 40 + 20 + 15 + 25 + 30 + 20 + 25 = 270$

$270 \div 10 = 27$

The mean practice time is 27 minutes.

Step 3 Predict the mean for the population.

The mean practice time for the sample is 27 minutes.

The mean for the population should be about 27 minutes.

Step 4 Decide if the prediction is reasonable.

The sample was a random sample.

The size of the sample (10) is fairly large compared to the population (60).

The prediction is reasonable.

Solution **The mean practice time for the sample is 27 minutes. The sample mean provides a reasonable prediction of the population's mean practice time.**

Example 2

Mindy is the captain of the dance team at her school. She is running in a class election for class president. April surveyed the students on the dance team to see whom they planned to vote for in the election. The results of her survey are shown below.

Student	Number of votes
Mindy	18
Tobey	6
Roland	7

Based on the survey, predict who will win the class election. Is the prediction reasonable?

Strategy **Use the data to make a prediction. Evaluate the data to decide if the prediction is reasonable.**

Step 1 Use the data to make a prediction.

From the data in the table, the student with the greatest number of votes is Mindy.

Based on the data, Mindy should win the election.

Step 2 Evaluate the data to decide if the prediction is reasonable.

Mindy has quite a few more votes than either of the other students in the table.

However, Mindy is the captain of the dance team. All of the students who were surveyed are on the dance team. This suggests that they might be biased toward Mindy.

The prediction that Mindy will win does not seem reasonable.

Solution **Although the results of the survey suggest that Mindy will win, the survey is biased. The prediction that Mindy will win is not reasonable.**

Lesson Practice

Choose the correct answer.

1. The heights of five pepper plants, in centimeters, selected at random from a greenhouse with 50 pepper plants are shown below.

   20, 24, 18, 23, 26

   Which is a reasonable prediction of the mean height of all the pepper plants in the nursery?

   A. 19 cm
   B. 22 cm
   C. 25 cm
   D. 26 cm

2. A city is having a clean-up day for the 25 parks in the city. The list below shows the ages of volunteers participating in the clean-up project at one of the parks.

   17, 18, 15, 16, 24, 20, 16

   Which is a reasonable prediction of the mean age of all the participants in the clean-up project?

   A. 16 years old
   B. 17 years old
   C. 18 years old
   D. 19 years old

3. Ines read the following number of pages each day last week: 76, 123, 84, 110, 36, 20, and 90. Which is a reasonable prediction of the mean number of pages she reads each day throughout the year?

   A. 75
   B. 85
   C. 95
   D. 100

4. Vincent stood by the log flume at an amusement park and asked 120 people exiting the ride to name their favorite ride. The table below shows the results of his survey.

Favorite Ride

Ride	Number of People
Roller coaster	25
Ferris wheel	22
Log flume	58
Carousel	15

Which of the following is the most reasonable prediction based on the survey?

A. The results of the survey that show the log flume is the favorite ride are biased results because only log flume riders were surveyed.
B. The results of the survey that show the log flume is the favorite ride are reasonable results.
C. The results of the survey that show the log flume is the favorite ride are unreasonable because the number of people surveyed was so small.
D. The results of the survey that show the roller coaster is the favorite ride are unbiased results.

Common Core State Standards:
7.SP.3, 7.SP.4

Compare Data Sets

Getting the Idea

You can compare the means, medians, modes, ranges, and interquartile ranges of two different data sets to draw conclusions about the data.

Example 1

Students at two middle schools sold reusable bags to raise money for Earth Day. Nellie surveyed a sample of ten students from each school to find out how many bags each student sold. Her data is shown in the stem-and-leaf plots.

Roosevelt Middle School

Stem	Leaves
1	0 3 7 9
2	0 1 4 4
3	5
4	2

Key: 1 | 0 = 10 bags

Madison Middle School

Stem	Leaves
1	2 4
2	1 3 6 8 9
3	0 0 4
4	

Key: 1 | 2 = 12 bags

Compare the measures of central tendency for the two schools.

What conclusions can you draw from the comparisons?

Strategy **Find the mean, median, and mode of each data set. Then compare.**

Step 1 Find the mean number of bags for Roosevelt Middle School.

The numbers of bags that students sold are listed below.

10, 13, 17, 19, 20, 21, 24, 24, 35, 42

$$\text{mean} = \frac{10 + 13 + 17 + 19 + 20 + 21 + 24 + 24 + 35 + 42}{10} = \frac{225}{10} = 22.5$$

Step 2 Find the median number of bags for Roosevelt Middle School.

The median is the middle number.

There are 10 values, so the median is the mean of the two middle values.

Find the mean of the fifth and sixth entries in the stem-and-leaf plot.

$$\text{median} = \frac{20 + 21}{2} = \frac{41}{2} = 20.5$$

Step 3 Find the mode number of bags for Roosevelt Middle School.

The mode is the value that occurs most often.

The mode is 24.

Step 4 Find the mean number of bags for Madison Middle School.

The numbers of bags that students sold are listed below.

12, 14, 21, 23, 26, 28, 29, 30, 30, 34

$$\text{mean} = \frac{12 + 14 + 21 + 23 + 26 + 28 + 29 + 30 + 30 + 34}{10} = \frac{247}{10} = 24.7$$

Step 5 Find the median number of bags for Madison Middle School.

Find the mean of the fifth and sixth entries in the stem-and-leaf plot.

$$\text{median} = \frac{26 + 28}{2} = \frac{54}{2} = 27$$

Step 6 Find the mode number of bags for Madison Middle School.

The mode is 30.

Step 7 Compare the measures of central tendency.

Compare the means: $24.7 - 22.5 = 2.2$

Compare the medians: $27 - 20.5 = 6.5$

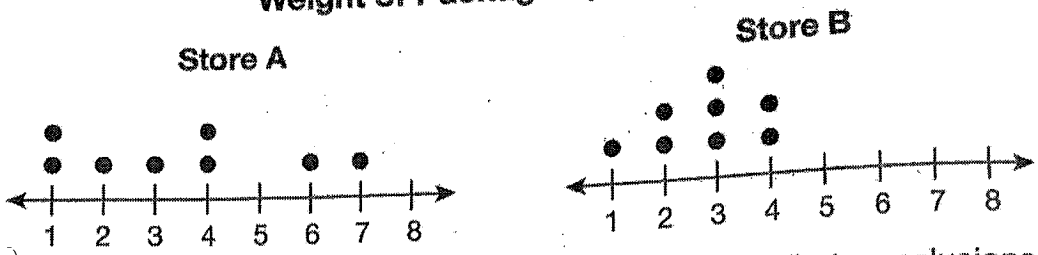
Compare the modes: $30 - 24 = 6$

Solution **Students at Madison Middle School sold an average of about 2 to 6 more bags than the students at Roosevelt Middle School.**

Example 2

The dot plots show the weights of 8 packages that are waiting to be shipped from two stores owned by a large shipping company.

Weight of Packages (in pounds)



Compare the measures of central tendency for the two stores. What conclusions can you draw from the comparisons? What conclusions can you draw from the dot plots?

Strategy **Find the mean, median, and mode of each data set. Then compare.**

Step 1 Find the mean, median, and mode for Store A.

The weights of the packages at Store A are 1, 1, 2, 3, 4, 4, 6, and 7.

$$\text{mean} = \frac{1+1+2+3+4+4+6+7}{8} = \frac{28}{8} = 3.5$$

$$\text{median} = \frac{3+4}{2} = \frac{7}{2} = 3.5$$

modes = 1 and 4

Step 2 Find the mean, median and mode for Store B.

The weights of the packages at Store B are 1, 2, 2, 3, 3, 3, 4, 4.

$$\text{mean} = \frac{1+2+2+3+3+3+4+4}{8} = \frac{22}{8} = 2.75$$

$$\text{median} = \frac{3+3}{2} = \frac{6}{2} = 3$$

mode = 3

Step 3 Compare the measures of central tendency.

Compare the means: $3.5 - 2.75 = 0.75$

Compare the medians: $3.5 - 3 = 0.5$

Compare the modes: You cannot compare the modes since one set of data has 1 mode and the other set has 2 modes.

The mean and median weights of the packages are 0.5 to 0.75 pounds heavier at Store A than at Store B.

Lesson Practice

Choose the correct answer.

Use the stem-and-leaf plots for questions 1–3.

The stem-and-leaf plots show the number of students who worked in the computer lab during the months of September and October.

September

Stem	Leaves
1	0 3 6 7 8
2	1 2 3 4 4 7
3	2 2 2 9

Key: 1 \| 0 = 10 students

October

Stem	Leaves
1	1 4 5 5 7 8 8 9
2	0 0 0 2 5 6
3	1

Key: 1 \| 1 = 11 students

1. Which statement about the median number of students in the computer lab is true?

   A. The median for September is the same as the median for October.
   B. The median for September is 4 more than the median for October.
   C. The median for September is 3 less than the median for October.
   D. The median for September is 2 more than the median for October.

2. Which statement about the mode number of students in the computer lab is true?

   A. The mode for September is the same as the mode for October.
   B. The mode for September is 4 less than the mode for October.
   C. The mode for September is 8 more than the mode for October.
   D. The mode for September is 12 more than the mode for October.

3. Which statement about the range of the number of students in the computer lab is true?

   A. The range for September is the same as the range for October.
   B. The range for September is 12 more than the range for October.
   C. The range for September is 9 more than the range for October.
   D. The range for September is 2 less than the range for October.

Domain 5: Cumulative Assessment for Lessons 28–35

1. Reyes conducted an experiment by tossing two number cubes a total of 72 times. He found the sums and recorded the results in the table.

Sum	Frequency
2	2
3	4
4	7
5	7
6	11
7	12
8	9
9	9
10	5
11	5
12	1

Which sum has a greater experimental probability than theoretical probability?

A. 3

B. 5

C. 6

D. 12

2. Isabelle is taking a survey to find the most popular music group of students in her community. Which of these is **not** a way for her to get a representative sample of this information?

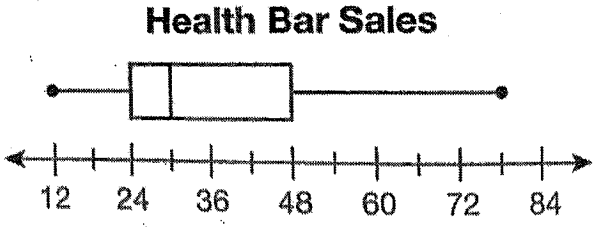
A. ask every tenth student she sees at a concert

B. ask every fifth student entering her school in the morning

C. ask every third student she encounters at the mall

D. ask every student at a local movie theater

3. Students on a soccer team sold health bars for a fund-raiser. The box plot shows the number of bars sold by the students on the team.



Which statement is **not** true about the data?

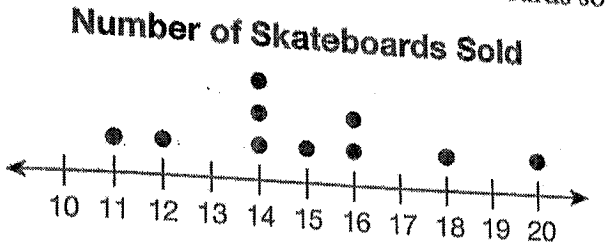
A. The range of the sales is 72 bars.

B. The first quartile of the data is 24 bars.

C. The third quartile of the data is 48 bars.

D. The interquartile range of the data is 24 bars.

4. The dot plot shows the number of skateboards sold at a shop each week over 10 weeks. The data are a representative sample of the number of skateboards sold throughout the year.



Which best represents the mean number of skateboards sold each week throughout the year?

A. 14
B. 14.5
C. 15
D. 15.5

5. The stem-and-leaf plots show the ages of people who attended a local play on two different nights.

Thursday

Stem	Leaves
1	8 9
2	0 3 5
3	0 1 4 6 7 9 9
4	3 4 7 8
5	2 5 6 8

Key: 1 | 8 = 18 years old

Friday

Stem	Leaves
1	1 2 2 3 4 5 6 6
2	1 2 4 5 6 7
3	0 2 5
4	1 6
5	3

Key: 1 | 1 = 11 years old

Which statement about the median ages is true?

A. The median for Thursday is the same as the median for Friday.
B. The median for Thursday is 15 more than the median for Friday.
C. The median for Thursday is 18 more than the median for Friday.
D. The median for Thursday is 20 more than the median for Friday.

Session 1

1. Ernie walks $\frac{1}{6}$ mile in $\frac{1}{12}$ hour when he walks along the river trail. How many miles per hour does Ernie walk when he hikes on the trail?

   A. $\frac{1}{72}$ mile per hour

   B. $\frac{1}{2}$ mile per hour

   C. 2 miles per hour

   D. 3 miles per hour

2. A submarine at an elevation of -35 feet rises 35 feet. What is the elevation of the submarine after it rises?

   A. -70 feet

   B. 0 feet

   C. 35 feet

   D. 70 feet

3. A map has a scale of $\frac{1}{2}$ inch $= 16$ miles. The distance between two parks on the map is $2\frac{3}{4}$ inches. How many miles apart are the parks?

   A. 40 miles

   B. 44 miles

   C. 76 miles

   D. 88 miles

4. A number cube labeled 1 to 6 is rolled 450 times. Which is the best prediction for the number of times that the number cube will land on a number less than 3?

   A. about 150 times

   B. exactly 150 times

   C. about 225 times

   D. exactly 225 times

5. Which is equivalent to the expression below?

   $6x + 2x + y + 3y$

   A. $2(4x + y)$

   B. $4(2x + y)$

   C. $4(2x + 2y)$

   D. $8(x + y)$

6. Tia generated a list of random numbers to simulate an experiment based on the waiting times at a supermarket checkout line. The number 1 represents waiting in line less than 5 minutes and the number 2 represents waiting in line 5 minutes or more.

   1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 2, 1

   Which conclusion can Tia most reasonably draw from the simulation?

   A. A person is likely to wait in line less than 5 minutes 14% of the time.

   B. A person is likely to wait in line 5 minutes or more 11% of the time.

   C. A person is likely to wait in line less than 5 minutes more than 50% of the time.

   D. A person is likely to wait in line 5 minutes or more 50% of the time.

7. There are 12 girls and 16 boys in a diving class. One student is randomly selected to dive first. What is the probability that the student selected to dive first is a girl?

   A. $\frac{3}{8}$

   B. $\frac{3}{7}$

   C. $\frac{4}{7}$

   D. $\frac{3}{4}$

8. A circular garden has a diameter of 6 feet. Which of the following is closest to the area of the garden?

   A. 18.84 square feet
   B. 28.26 square feet
   C. 37.68 square feet
   D. 113.04 square feet

9. Jerome earns $200 per month plus $12 for each bike he sells at the bike shop. This month, he wants to earn at least $400. What is the least number of bikes that Jerome can sell to reach his goal?

   A. 15
   B. 16
   C. 17
   D. 33

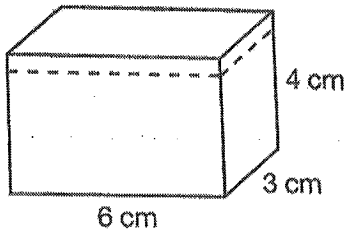
10. How is $4\frac{3}{11}$ written as a decimal number?

   A. $4.\overline{27}$
   B. 4.272
   C. 4.311
   D. $4.3\overline{6}$

11. Which of the following equations represents a proportional relationship?

   A. $\frac{3}{2} = \frac{15}{12}$
   B. $\frac{8}{3} = \frac{36}{14}$
   C. $\frac{6}{9} = \frac{24}{32}$
   D. $\frac{4}{8} = \frac{15}{30}$

12. A piece of cheese has the dimensions of the rectangular prism shown below.



Marina cuts a slice of the cheese by making a horizontal cut across the top along the dotted lines. Which best describes the shape and dimensions of the top face of the slice?

   A. square with side lengths 4 cm
   B. rectangle with sides 6 cm by 3 cm
   C. rectangle with sides 6 cm by 4 cm
   D. rectangle with sides 3 cm by 4 cm

13. A weather balloon at an elevation of 344 feet descends at a constant rate. The balloon reaches the ground in 4 minutes. Which best describes the rate at which the balloon's elevation changed?

   A. −86 feet per minute
   B. −84 feet per minute
   C. 84 feet per minute
   D. 86 feet per minute